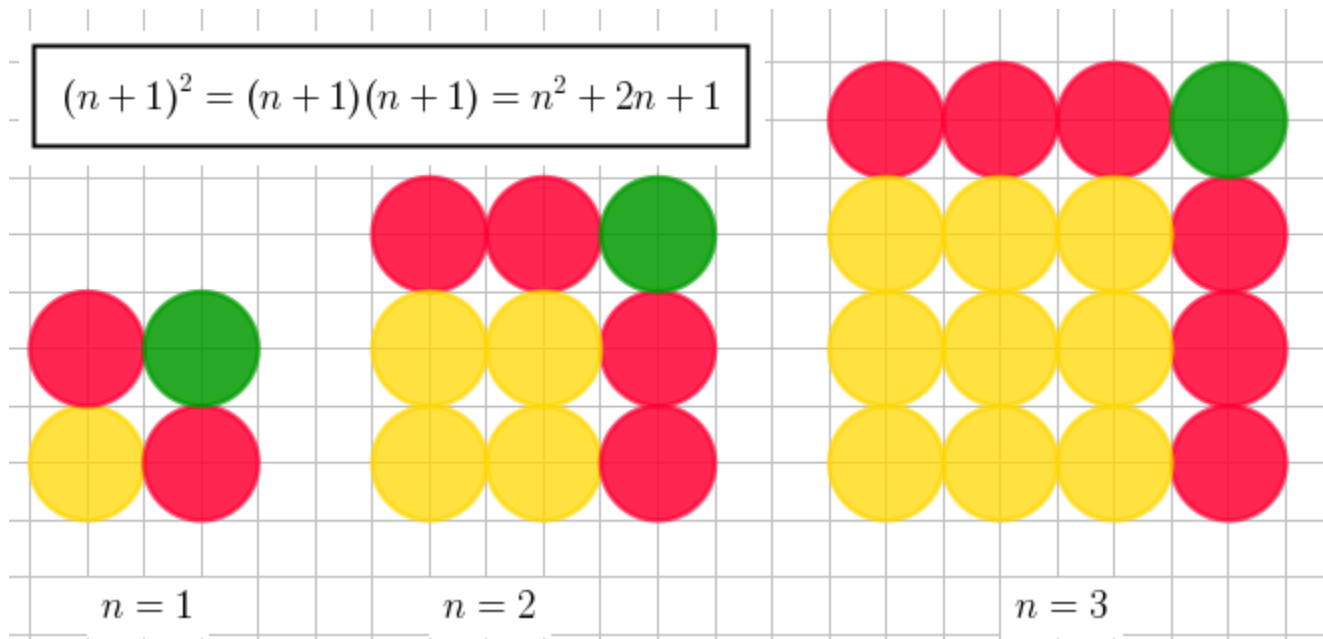


Grade 10 FMP

RBSS Fall

Unit 3: Factors and Products



Name:

Topic
Substitution
Exponent laws review & negative exponents
Expand brackets with exponents
Factor with exponents
Expand two or more brackets
Factor a trinomial
Factor a difference of two squares
Project

Vocabulary

Integer	A whole number. An integer can be positive, negative or zero. The set of integers has a math symbol that looks like a 'z': \mathbb{Z}
Prime number	A number with exactly two factors: 1 and itself. The first few prime numbers are: 2, 3, 5, 7, 11, 13, 17, 19, 23,
Factor	A number that divides a given integer. Eg, 12 has six factors: 1, 2, 3, 4, 6, 12
Prime factors	The factors of a number that are prime numbers. The prime factors of 12 are 2 and 3.
Prime factorization	To write a number as a product of prime numbers only. The prime factorization of 12 is $12 = 2 \times 2 \times 3$. Every whole number has a unique prime factorization.
Multiple	The multiples of 5 are 5, 10, 15, 20, ... etc. The third positive multiple of 5 is 15.
Common factor	Any number that divides two or more given numbers. Eg, the common factors of 12 and 20 are 1, 2, 4. There are only three common factors of 12 and 20.
HCF	Highest Common Factor: $HCF(12,20)=4$ (also known as GCD greatest common divisor)
Common multiple	Any numbers that appear in the multiplication tables of two or more given numbers. Eg, the common multiples of 6 and 10 are: 30, 60, 90, ... There are infinitely many common multiples of 6 and 10.
LCM	Least Common Multiple: $LCM(6,10)=30$
To expand brackets	Use the distributive property to rewrite an expression given in brackets. Eg, $3(4x + 5) = 12x + 15$
To factor	To reverse expanding brackets. To write the sum of terms as a product of terms, eg $12x + 15 = 3(4x + 5)$
Equivalent	The same expression, written in two or more different ways. $12x + 15$ is equivalent to $3(4x + 5)$.
To factor fully	After factoring fully, there should be no common factor left within any one bracket.
Monomial	An expression that has only one term. Any exponents must be positive integers.
Binomial	An expression that has two terms. Any exponents must be positive integers.
Trinomial	An expression that has three terms. Any exponents must be positive integers.

Summary of Exponent Laws

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$a^{-n} = \frac{1}{a^n}$$

$$a^0 = 1$$

$$(a^m)^n = a^{mn}$$

$$(a \cdot b)^n = a^n \times b^n$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

Multiples chart:

	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

Unit 3 Factors and Products

Lesson 1: Practice multiples & substitution

Multiples: Learning math is easier when the first 12 multiplication tables are familiar.

Complete at least one of these multiplication tables:

	2	3	4	5	6	7	8	9	10
8									
4									
10									
7									
9									

	2	3	4	5	6	7	8	9	10
7									
6									
15									
9									
8									

	3	2	5	7	4	6	9	10	8
6									
8									
9									
20									
7									

Substitution

Video 20 on www.corbettmaths.com

Question 1: If $a = 7$ $b = 10$ $c = 3$ $d = 8$ and $e = 15$
Find the value of each expression.

(a) $a + 5$

(d) $e - d$

(e) $2a$

(h) $5c$

(i) $\frac{b}{2}$

(l) $\frac{a}{2}$

(m) a^2

(p) d^2

(q) $2a + 1$

(t) $4e - 45$

(u) $2a + 3c$

(x) $e - 4c$

Question 2: If $f = 5$ $g = 6$ $h = 4$ and $i = 2$
Find the value of each expression.

(a) fg

(c) fgh

(e) \sqrt{h}

(g) $5h + 7i$

Question 3: If $a = -2$ $b = 5$ $c = -6$ $d = 10$ and $e = 9$
Find the value of each expression.

(a) $a + 4$

(d) $a - d$

(e) $d - c$

(h) $-7b$

(i) $2d + 3c$

(l) $20 + 4a$

(m) ac

(p) $bd + a$

(q) $\frac{a}{2}$

(t) c^2

Question 4: If $a = 1.5$ $b = 4$ $c = 6$ $d = 0.5$ and $e = -3$
Find the value of each expression.

(a) $4(a + d)$

(c) $3(10 - e)$

(e) e^3

(g) $5b^2$

(i) $\frac{b+2}{3}$

(k) $\frac{10d+4b}{7}$

(l) $b^2 + 5b - 5$

(m) $5e^2 - 4e + 7$

Let $x = 4$. Create an expression with the letter x that is worth exactly 100. Eg $4\sqrt{6x+1} + 5x^2$	Choose a value for y . Let $y =$ Create an expression with the letter y that is worth exactly -1 and has at least three terms.
---------------------------------------------------------------------------------------------------------------	-----------------------------------------------------------------------------------------------------------------------------------------

Complete the heading row and column only of this multiplication table.

You must use the numbers 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 15, 20 once each on the heading row and the heading column.

	80												40
		8											80
			84									63	
				300						140			
					99				27				
						55		66					
							150						
					15			18					
					88				24				
				30						14			
			72								54		
		24										240	
	40												20

Unit 3 Factors and Products

Lesson 2: Practice exponent laws & substitution

Complete this exponent table:

x	2^x	3^x	4^x	5^x	8^x	9^x	10^x
-1	$\frac{1}{2}$						
0							
1	2	3	4	5			10
2		9					
3	8			125			
4							
5	32						
6							100000
7	128						
8							
9							
10							



Draw a line to match each exponent calculation with its simplification:

$3^5 \times 3^7$									
	5^7	$7^4 \times 7^2$		$7^5 \div 7^6$	5^{-3}	$(3^5)^7$			
	$\frac{5^7}{5}$		$(5^4)^3$				$3^7 \div 3^7$		
$\frac{1}{5^3}$									
	5^{12}	7^6	3^0	5^6	3^{35}	3^{12}		$\frac{1}{7}$	

Exponent laws:

Name of law	Calculation	Means....	Single Power
Product	$x^3 \times x^5 =$	$(x \times x \times x) \times (x \times x \times x \times x \times x)$	$= x^8$
Quotient	$p^6 \div p^2 =$	$\frac{p \times p \times p \times p \times p \times p}{p \times p}$	$= p^4$
Power	$(a^2)^4 =$	$(a \times a) \times (a \times a) \times (a \times a) \times (a \times a)$	$= a^8$

Rewrite the following as a single exponent:

$9^3 \times 9^7 =$	$a^3 \times a^{-5} =$	Simplify (show your work): $\frac{(7^3 \times 7^5)^2}{7^{10} \times 7^4}$
$(7^4)^5 =$	$x^2 \div x^{-8} =$	
$\frac{8^7}{8^2} =$	$(y^4)^7 =$	

Draw a line to match each exponent calculation with its simplification:

$a^3 \times a^7$ $p^4 \times p^2$ $g^8 \div g^2$ $(g^8)^3$
 $\frac{g^{13}}{g^2}$ $(a^5)^3$ p^{-2} $a^4 \div a$
 g^6 p^6 g^{24} a^{10} $\frac{1}{p^2}$ g^{11}
 a^{15} a^3

Substitute the given value for x into each expression and evaluate. If the answer is not an integer, round your answer correct to 2 decimal places.

$4x^3 + 7x^2 + x + 8$ $x = 10$	$\sqrt{x^6 - 100}$ $x = 4$
$\frac{5x + 6}{3}$ $x = 12$	$\left(\frac{3x + 8}{7} + \frac{20 - x}{3}\right)^2$ $x = 2$

Use a mini whiteboard/notepaper for your working. Simplify each expression:

1) $a \cdot a^2 \cdot a^3$ 2) $(2a^2b)(4ab^2)$ 3) $(6x^2)(-3x^5)$ 4) $b^3 \cdot b^4 \cdot b^7 \cdot b$ 5) $(3x^3)(3x^4)(-3x^2)$

6) $(2x^2y^3)^2$ 7) $(5x^2y^4)^3$ 8) $(6x^4y^6)^3$ 9) $(4x^3y^3)^3$ 10) $(7xy)^2$

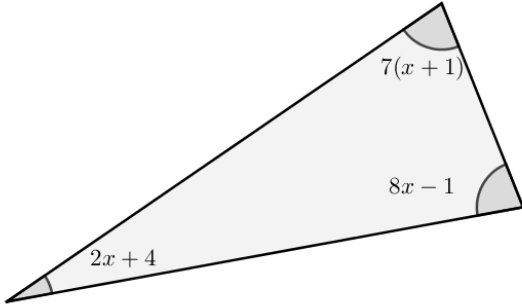
11) $\frac{x^3}{x}$ 12) $\frac{18c^3}{-3c^2}$ 13) $\frac{9a^3b^5}{-3ab^2}$ 14) $\frac{-48c^2d^4}{-8cd}$ 15) $\frac{22y^6z^8}{2yz^{-7}}$

16) $x^2 \cdot x^7$ 17) $(x^2)^7$ 18) $(-2x^4)^5$ 19) $2x^3 + 7x^3$ 20) 7^0

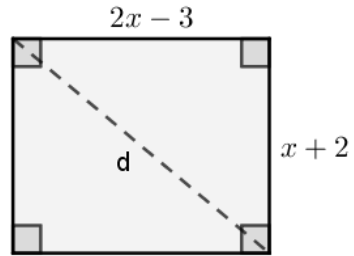
21) $8x^0$ 22) -3^4 23) $(-3)^4$ 24) $6x^0y^8 - (2y^2)^4$ 25) $(x + 2y)(x - 2y)$

26) $\frac{2x^3}{-8x^4}$ 27) $\frac{xy^7}{x^3y^4}$ 28) $6x^5 \cdot 3x^5 \cdot x^0$ 29) $(3st^{12})^3$ 30) $\left(\frac{3m^2n^7}{m}\right)^5$

Calculate the size of each angle.

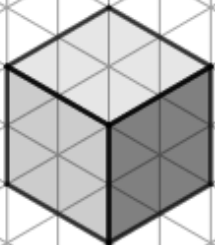
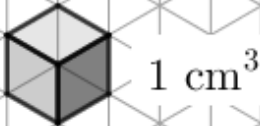


The perimeter of this rectangle is 31 cm. Calculate the length of the diagonal d .



A 1 cm^3 cube is drawn below alongside another cube with side length 2. There are 8 of the little cubes in the cube with side length 2.

Draw a cube that contains 125 of the little 1 cm^3 cubes.



Lesson 3: Exponents & Expanding Brackets

Division practice (no calculator)

 $1. \quad 30 \div 6 =$

$2. \quad 27 \div 9 =$

$3. \quad 49 \div 7 =$

$4. \quad 22 \div 11 =$

$5. \quad 96 \div 8 =$

$6. \quad 16 \div 8 =$

$7. \quad 99 \div 11 =$

$8. \quad 9 \div 3 =$

$9. \quad 30 \div 3 =$

$10. \quad 24 \div 6 =$

$11. \quad 55 \div 5 =$

$12. \quad 24 \div 4 =$

$13. \quad 16 \div 4 =$

$14. \quad 35 \div 5 =$

$15. \quad 36 \div 4 =$

$16. \quad 66 \div 11 =$

$17. \quad 84 \div 7 =$

$18. \quad 40 \div 8 =$

$19. \quad 21 \div 7 =$

$20. \quad 54 \div 9 =$

$21. \quad 72 \div 6 =$

$22. \quad 108 \div 9 =$

$23. \quad 21 \div 3 =$

$24. \quad 20 \div 5 =$

 $25. \quad 56 \div 7 =$

$26. \quad 66 \div 11 =$

$27. \quad 45 \div 9 =$

$28. \quad 121 \div 11 =$

$29. \quad 72 \div 8 =$

$30. \quad 84 \div 7 =$

$31. \quad 14 \div 7 =$

$32. \quad 12 \div 4 =$

$33. \quad 15 \div 5 =$

$34. \quad 16 \div 8 =$

$35. \quad 36 \div 6 =$

$36. \quad 22 \div 11 =$

$37. \quad 64 \div 8 =$

$38. \quad 81 \div 9 =$

$39. \quad 66 \div 6 =$

$40. \quad 25 \div 5 =$

$41. \quad 50 \div 5 =$

$42. \quad 3 \div 3 =$

$43. \quad 18 \div 6 =$

$44. \quad 27 \div 9 =$

$45. \quad 32 \div 4 =$

$46. \quad 15 \div 3 =$

$47. \quad 30 \div 3 =$

$48. \quad 48 \div 4 =$

Expand the brackets:	Fill in the brackets ('Factor'):
1. $5(2x - 4) =$	1. $15x + 20 = 5(\quad)$
2. $7(3a + b) =$	2. $6x + 21 = 3(\quad)$
3. $3(2p + 5q) =$	3. $4x + 20 = 4(\quad)$
4. $9(2x + 1) =$	4. $25p + 30q = 5(\quad)$
5. $3(a^2 - b^2) =$	5. $18x + 24y + 18z = 6(\quad)$
6. $12(3p + 5q) =$	6. $51a + 17b + 34c = 17(\quad)$



Expand the brackets:

1. $3x(x^2 - 5) =$
2. $5a(a + 1) =$
3. $12p^3(p^2 - 1) =$
4. $-5x^4(5 - x) =$
5. $3a^2b(2a + 5b) =$
6. $5a(a^3 + a^2 + a + 1) =$
7. $5p^2\left(6 + \frac{3}{p} + p^3\right) =$
8. $3x^5\left(4x^2 + 7x + \frac{3}{x}\right) =$

Fill in the term in front of the brackets:



$$(3x^4 - 5) = 6x^5 - 10x$$

$$(a^2 + 1) = 7a^4 + 7a^2$$

$$(p^3 - 2) = 5p^7 - 10p^4$$

$$(7 - 2x) = 21 - 6x$$

$$(5a + 2b) = 35a^2b + 14ab^2$$

$$(a^3 + a^2 + a + 1) = a^{13} + a^{12} + a^{11} + a^{10}$$

Fill in the missing terms:

$$(2x^3 - \quad) = 14x^4 - 77x$$

$$(a^2 + 1) = -2a^3 - \quad$$

$$(7p^5 - \quad) = 21p^6q^2 - 6pq^3$$

$$6x(\quad - \quad) = 30x - 66x^2$$

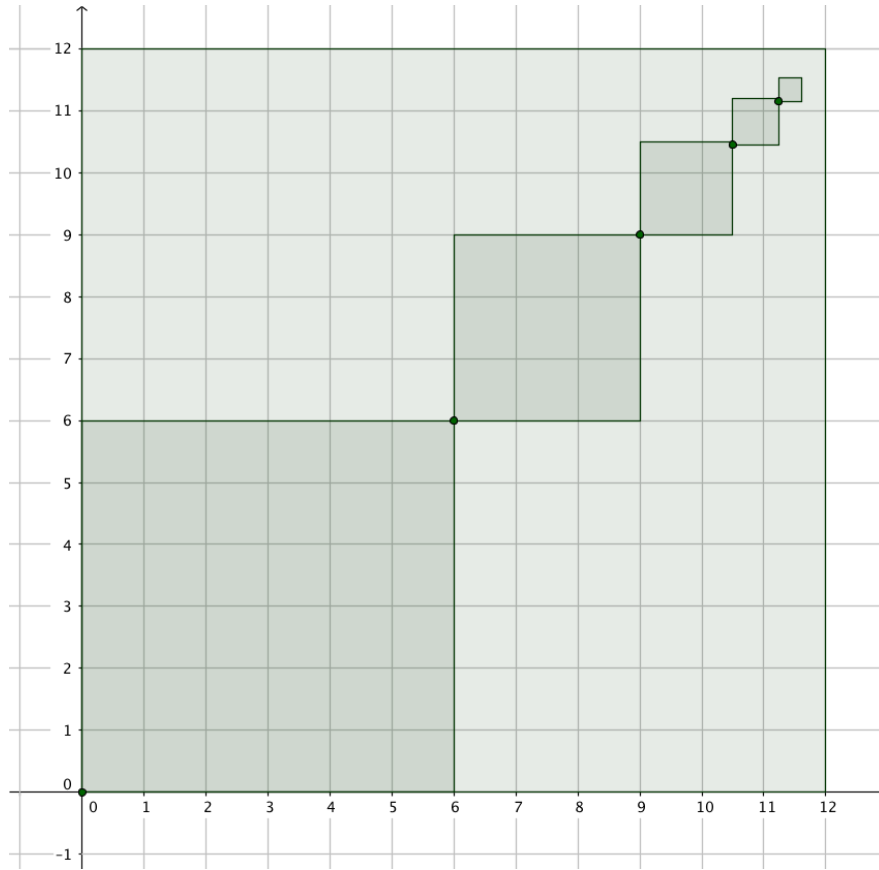
$$3ab^2(\quad + \quad) = 6a^2b^2 + 15ab^3$$

$$3a^4(4a^3 + \quad + 2a + \quad) = \quad + 9a^6 + \quad + 3a^4$$

$$(\quad + \quad - \quad) = 30x^3 + 15x^2 - 10x$$



Sequence of Squares:



Complete the table. Use **simplified fractions**, not decimals.

Square # (n)	Side length (l)	Area (A)
1	6	36
2	3	9
3	$\frac{3}{2}$	$\frac{9}{4}$
4		
5		
10		
n	$l = 12 \times \left(\frac{1}{2}\right)^n$	A =

After 3 squares, the sum of side lengths is $6 + 3 + \frac{3}{2} = \frac{21}{2} = 10.5$. After 5 squares, how close is the sum of side lengths to 12? That is, how close is the sequence of squares to the top right corner?

Unit 3 Factors and Products

Lesson 4: Factor a polynomial with one bracket

Factors of whole numbers. The last pair in a list factors is the factor pair that is or is closest to \sqrt{n} . When we list pairs up to the square root, we have listed all the pairs.

A. List all the factor pairs of these numbers:

36	24	60	100
1×36	1×24	\times	\times
$2 \times$	$2 \times$	\times	\times
\times	\times	\times	\times
\times	\times	\times	\times
6×6		\times	\times
		\times	
$(\sqrt{36} = 6)$	$(\sqrt{24} = 4.898 \dots)$	$\sqrt{60} = 7.745 \dots$	$\sqrt{100} = 10$
18	25	12	13
\times	\times	\times	\times
\times	\times	\times	
\times		\times	
30	50	40	80
\times	\times	\times	\times
\times	\times	\times	\times
\times	\times	\times	\times
\times		\times	\times
			\times

Factoring with one bracket

B. Examples

$$7x - 21$$

$$= 7 \cdot x - 7 \cdot 3$$

$$= 7(\quad)$$

$$15x + 20$$

$$12x + 16y$$

$$4x^2 + 12x$$

$$4xyz + 6xy$$

$$30a^2b^3 - 48a^3b^2$$

$$= 6 \cdot 5 \cdot a \cdot a \cdot b \cdot b \cdot b - 6 \cdot 8 \cdot a \cdot a \cdot a \cdot b \cdot b$$

$$=$$

$$-8x^4 + 4x^3 - 2x^2$$

$$-12p^3q^2 + 8p^2q - 16pq^5$$

C: Linear expressions

1. HCF(36,24) = 12	$36x + 24$ $= 12(\quad)$
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2. HCF(18,9) = 9	$18x + 9$ $= 9(\quad)$
------------------	-----------------------------

3. HCF(27,54) = 27	$-27x - 54$ $= -27(\quad)$
--------------------	---------------------------------

4. HCF(12,12) =	$-12x + 12$ $= -12(\quad)$
-----------------	---------------------------------

5. HCF(12,8) =	$12x + 8$ $=$
----------------	------------------

6. HCF(45,35) =	$-45x + 35$ $=$
-----------------	--------------------

7. HCF(16,12) =	$-16x - 12$ $=$
-----------------	--------------------

8. HCF(25,45) =	$-25x + 45$ $=$
-----------------	--------------------

D. Factoring with an exponent

1. $\text{HCF}(20x^5, 16x^3) = 4x^3$

$20x^5 + 16x^3 = 4x^3(\quad)$

2. $\text{HCF}(5y^5, 15y^2) =$

$5y^5 + 15y^2 =$

3. $\text{HCF}(9p^2, 9p^5) =$

$9p^5 - 9p^2 =$

4. $\text{HCF}(72a^5, 9a^4) = 9a^4$

$-72a^5 + 9a^4 = -9a^4(\quad)$

5. $\text{HCF}(30b^5, 12b^3) =$

$-30b^5 - 12b^3 =$

6. $10a^4 + 2a =$

7. $70b^5 - 10b^2 =$

8. $-5c^2 - 15c =$

9. $-10d^4 + 50d =$

10. $9e^4 + 90e =$



E. Factoring with two exponents

1. $-6a^3b^4 - 54a^3b^2$
=

2. $-3c^3d - 21cd$
=

3. $6p^5q^5 + 54p^3$
=

4. $-70a^5b^4 - 10a^4b^5$
=

5. $12x^5y^6 - 32xy^3 =$
=

6. $20x^2y + 30y^4$
=

7. $3p^5q^2 + 5pq^4 =$
=

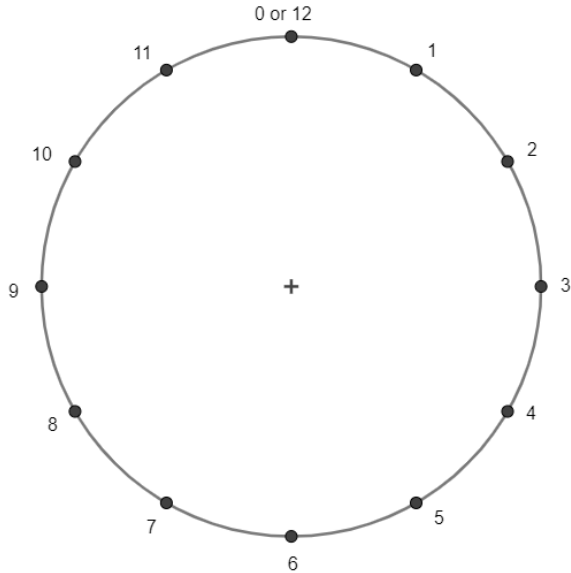
8. $-20a^4b^4 - 48ab^3$
=

9. $20c^3d^4 + 24c^2d^5 - 28cd^6$
=

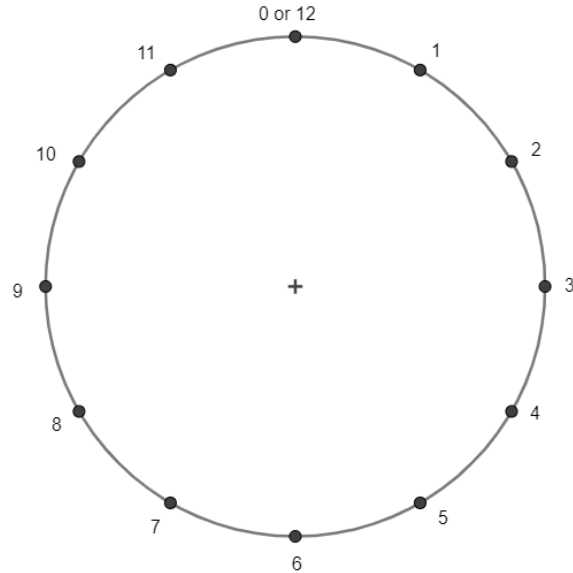
10. $60xyz + 66x^2y^3z^4 + 72x^2yz$
=

F. Use a ruler. Start at the zero dot. All increments give a pattern.

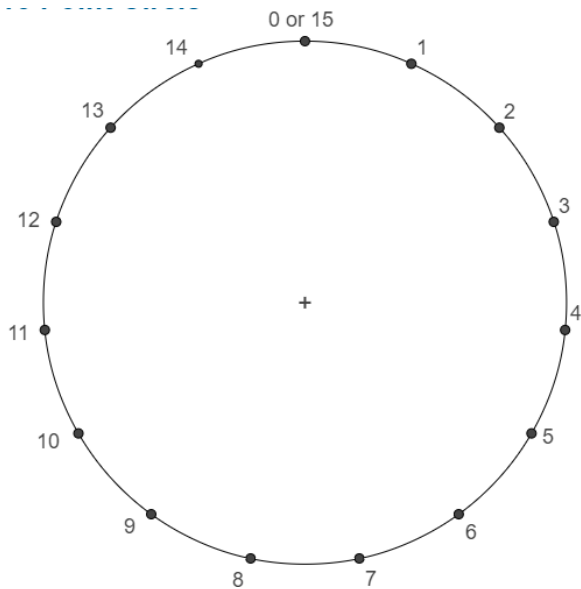
12 dots. Join the dots with an increment of 2.
(eg, 0 to 2; 2 to 4; 4 to 6 etc)



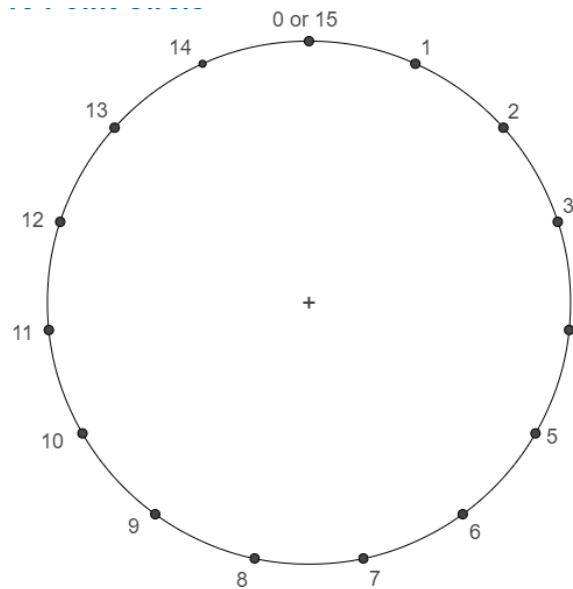
12 dots. Join the dots with an increment of 5.
(eg, 0 to 5; 5 to 10; 10 to 3 etc)



15 dots. Join the dots with an increment of 3.



15 dots. Join the dots with an increment of 4.



What is the result when the increment is a factor of the number of dots?

What is the result when the increment is not a factor of the number of dots?

Lesson 5: Expand two (or more) brackets

A. Complete these multiplications using any non-calculator method.

$$3 \times 76$$

$$3 \times 264$$

$$23 \times 76$$

$$34 \times 91$$

$$3 \times 23 \times 76$$

$$3 \times 5 \times 10$$

B. Multiplication with brackets – numerical values only:

$3 \times 76 =$

$3 \times 264 =$

$23 \times 76 =$

$34 \times 91 =$

$3 \times 23 \times 76 =$

Expanding more than one bracket

23×76

Brackets:

23×76

Grid:

	70	6
20		
3		

23×76

Brackets again (FOIL):

Expanding more than one algebraic bracket – general method:

$(3a + 8)(2a + 7)$	$(2x - 5)(x + 9)$
$3(4b + 5)(3b - 1)$	$(x + 3)(x + 4)(x - 5)$

Expanding binomial by binomial:

<p>METHOD 1: FOIL</p> <p>FIRST, OUTSIDE, INSIDE, LAST</p>	<p>METHOD 2: GRID</p>
-----------------------------------------------------------	-----------------------

Expand and simplify. Use either the foil method or the grid method on a whiteboard/notepaper.

$$(x + 2)(x + 6)$$

first

$$(x + 2)(x + 6)$$

outside

$$(x + 2)(x + 6)$$

inside

$$(x + 2)(x + 6)$$

last

1. $(x + 3)(x + 5)$

2. $(2x + 3)(x + 5)$

3. $(2x - 3)(x - 5)$

4. $(2x - 3)(x + 5)$

5. $(5x + 3)(x - 1)$

6. $(5x - 3)(x + 1)$

7. $(5x - 3)(x - 1)$

8. $(x - 4)(x + 5)$

9. $(x + 4)(x - 5)$

10. $(x - 4)(x - 5)$

11. $(x + 4)(x + 5)$

12. $(3a - 4)(5a - 4)$

13. $(7b + 2)(b - 2)$

14. $(n - 1)^2 + 4n$

15. $(n + 2)^2 + 4(n - 1)$

16. $(x - 4)(x + 4)$

17. $(x - 1)(x + 1)$

18. $(x - y)(x + y)$

19. $(a - b)(a + b)$

20. $(2x - 5)(2x + 5)$

21. $(x + 7)^2$
 $= (x + 7)(x + 7)$
 $=$

22. $(2x - 1)^2$

23. $(3x + 5y)^2$

24. $(x - 9)^2$

25. $(a + b)^2$

26. $(x - 5)(x -)$
 $= x^2 - 11x +$

27. $(x + 4)(x +)$
 $= x^2 + 11x +$

28. $(x +)^2$
 $= x^2 + 8x + 16$

29. $(x +)^2$
 $= (x +)(x +)$
 $= x^2 + + 25$

30. $(2x + 1)()$
 $= 10x^2 - x - 3$

Unit 3 Factors and Products

Lesson 6: Factor with two brackets ($a = 1$)

In this lesson we learn how to reverse the process of multiplying out brackets.

The grade 10 curriculum requires that you can reverse a process like this:

$$(x + 4)(x - 5) = x^2 - x - 20$$

That is, factor a trinomial of the form $x^2 + bx + c$.

The grade 11 curriculum requires that you factor trinomials of the form $ax^2 + bx + c$.

There are many techniques (split the middle technique, grid technique amongst others). All techniques require that you can solve an arithmetic puzzle such as these:

Find two numbers that multiply to and add to

$$\dots \times \dots = 45$$

$$\dots + \dots = 14$$

$$\dots \times \dots = 36$$

$$\dots + \dots = 15$$

$$\dots \times \dots = -40$$

$$\dots + \dots = 6$$

$$\dots \times \dots = 25$$

$$\dots + \dots = -10$$

$$\dots \times \dots = 12$$

$$\dots + \dots = -7$$

$$\dots \times \dots = -20$$

$$\dots + \dots = 1$$

$$\dots \times \dots = 36$$

$$\dots + \dots = 12$$

$$\dots \times \dots = -14$$

$$\dots + \dots = -5$$

Examples: Simple, Difference of Two Squares, Common Factor or negative.

$$x^2 + 14x + 45$$

$$x^2 + 12x + 36$$

$$x^2 - 10x + 25$$

$$x^2 - 5x - 14$$

$$x^2 - 9$$

$$16a^2 - 25b^2$$

$$3x^2 - 30x + 75$$

$$-x^2 + 5x + 14$$

Question 1: Factorise each of the following

(a) $x^2 + 7x + 12$ (b) $x^2 + 6x + 8$ (c) $x^2 + 5x + 6$ (d) $x^2 + 8x + 7$

(i) $x^2 + 10x + 25$ (j) $x^2 + 12x + 20$ (k) $x^2 + 25x + 24$ (l) $x^2 + 11x + 24$

Question 2: Factorise each of the following

(a) $x^2 + x - 12$ (b) $x^2 + 5x - 6$ (c) $x^2 + 3x - 10$ (d) $x^2 + 3x - 4$

Question 3: Factorise each of the following

(a) $x^2 - 3x - 10$ (b) $x^2 - x - 20$ (c) $x^2 - 6x - 27$ (d) $x^2 - 2x - 3$

Question 4: Factorise each of the following

(a) $x^2 - 6x + 9$ (b) $x^2 - 9x + 20$ (c) $x^2 - 9x + 14$ (d) $x^2 - 13x + 22$

Question 5: Factorise each of the following

(a) $x^2 - 9x + 8$ (b) $x^2 + 24x + 23$ (c) $x^2 - 5x - 14$ (d) $x^2 - 7x + 12$

(i) $x^2 - 11x + 30$ (j) $x^2 - 4x - 32$ (k) $x^2 - 2x - 63$ (l) $x^2 - 16x - 17$

Question 1: Factorise each of the following

- (a) $x^2 - 25$ (b) $y^2 - 49$ (c) $w^2 - 100$ (d) $x^2 - 4$
- (i) $16 - x^2$ (j) $1 - y^2$ (k) $81 - x^2$ (l) $144 - h^2$
- (q) $49x^2 - 16$ (r) $100 - 81x^2$ (s) $9x^2 - 4y^2$ (t) $36a^2 - c^2$

Question 2: Factorise **fully** each of the following

- (a) $2x^2 - 32$ (b) $2y^2 - 18$ (c) $2x^2 - 200$ (d) $3x^2 - 75$

Question 1: A quadratic expression, $x^2 + ax + 20$, can be factorised.
Find all possible values for a.
a can be positive or negative.

Question 2: A quadratic expression, $x^2 + bx + 16$, can be factorised.
Find all possible values for b.
b can be positive or negative.

Question 3: A quadratic expression, $x^2 - 6x + c$, can be factorised.
Find three possible values for c.

Factor fully:

$$2x^2 + 2x - 24$$

$$3x^3 + 21x^2 + 30x$$

$$x^2 - 16x + 63$$

$$x^2 - 2x$$

$$4x^2 + 16x + 12$$

$$5x^2 + 15x - 140$$

$$4x^2 - 9$$

$$3x^2 - 12$$

$$4x^2 - 100$$

$$3x^2 - 12x$$

$$5x^2 - 5$$

$$x^2 - 2x - 48$$

$$x^3 - 7x^2 - 8x$$

$$5x^2 - 80$$

$$2x^2 - 72$$

$$x^7 - 16x^6 + 63x^5$$

$$x^2 - 13x + 40$$

$$3x^2 - 36x + 105$$

Unit 3 Factors and Products

Lesson 7: Prime Factor Trees

Review Page:

Evaluate, don't use a calculator:

$2^3 =$	$3^3 =$	$4^3 =$	$5^3 =$	$6^3 =$	$10^3 =$
$2^4 =$	$3^4 =$	$4^4 =$	$5^4 =$		$10^4 =$

Simplify:

$x^5 \cdot x^3$	$(x^5)^3$	$\frac{p^7}{p^3}$	$(3a^3b^4)^2$	$(3a^3b^4)^2(9ab)$	$\frac{(3a^3b^4)^2}{9ab}$
-----------------	-----------	-------------------	---------------	--------------------	---------------------------

Expand the brackets (and simplify if possible):

$5(2x - 7)$	$3p^2(5p^3 + 2p^2 - 7p - 1)$
$(4x - 3)(5x + 1)$	$(x + 7)^2$
$(x - 8)(x + 8)$	$(2p - 3)(5p^2 + 3p - 2)$ $= 2p(5p^2 + 3p - 2) - 3(5p^2 + 3p - 2)$ $=$

Factor:

$15a + 25$	$15b^5 + 25b^2$
$6a^2b^3 + 10a^3b^4$	$x^2 + 9x + 20$
$2x^2 + 2x - 60$	$x^2 - 25$

Prime Factor Trees

The first 10 prime numbers are

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Writing a number as a product of primes is a bit like dissecting the number.

Examples:

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Prime factors can be used to find the HCF (Highest common factor)

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Prime factors can also be used to find the LCM (Lowest common multiple), however if you know the HCF you can also use a simple formula.

METHOD 1	METHOD 2 $\text{LCM}(m, n) = \frac{m \times n}{\text{HCF}(m, n)}$
----------	----------------------------------------------------------------------

Draw a prime factor tree on a whiteboard and write these numbers as a product of primes:

200 =	75 =
215 =	108 =
100 =	60 =
36 =	48 =

Use the prime factor method to find the HCF(m,n). Show your working. Use either method 1, method 2 or your own method to compute the LCM(m,n).

$$m = 200, n = 75$$

$$\text{HCF}(200,75)=$$

$$\text{LCM}(200,75)=$$

$$m = 200, n = 215$$

$$\text{HCF}(200,215)=$$

$$\text{LCM}(200,215)=$$

$$m = 200, n = 100$$

$$\text{HCF}(100,200)=$$

$$\text{LCM}(100,200)=$$

$$m = 36, n = 60$$

$$\text{HCF}(36,60)=$$

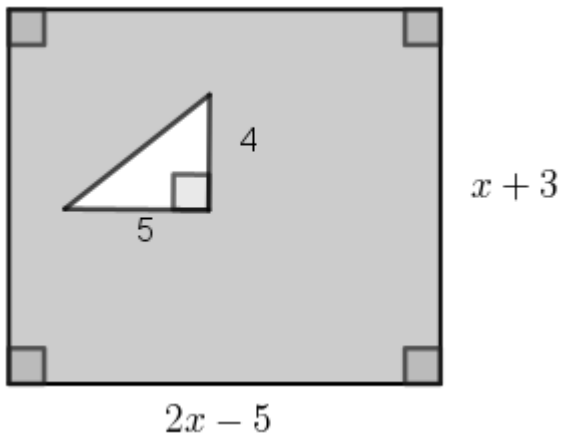
$$\text{LCM}(36,60)=$$

$$m = 36, n = 60, p = 40$$

$$\text{HCF}(36, 60, 40) =$$

$$\text{LCM}(36, 60, 40) =$$

GEOMETRY/ALGEBRA problem



What mathematical properties do you notice about the shape on the left?

What calculations are required to find the shaded area?

Show that the shaded area shaded in this shape can be calculated with the formula

$$A = 2x^2 + x - 25$$

Given that the the side labelled $2x - 5$ measures 15 cm, calculate the value of x .

Calculate the area shaded.

3. Substitute the value $a = 5$ and $b = -2$ into these expressions:

$$a + b = \qquad a - b = \qquad \frac{4a+1}{a+2} =$$

$$5b = \qquad b^2 = \qquad b^3 =$$

$$3ab = \qquad 3b^2 = \qquad b^3 - 3b^2 + 5b =$$

4. The trinomial $x^2 + 2x + 7$ has the value 127 when $x = 10$:

$$(10)^2 + 2(10) + 7 = 100 + 20 + 7 = 127$$

Find the value of the trinomial $x^2 + 2x + 7$ when

$$x = 0$$

$$x = 1$$

$$x = 5$$

$$x = -5$$

Section 2: Laws of exponents

5. Simplify:

$$x^9 \cdot x^{-6}$$

$$\frac{p^5}{p^{-3}}$$

$$(2a^5b^4)^3(3ab)^2$$

$$(5x^2)^3$$

$$(2a^5b^4)^3$$

$$\frac{(2a^5b^4)^3}{4a^9b^{13}}$$

Section 3: Expanding brackets

3. Multiply out the brackets and simplify:

$5(3x - 7) =$	$3a^2(4a^2 - 3a + 1) =$
$(b + 3)(b + 9) =$	$(d - 10)^2 =$
$(x - 8)(x + 5) =$	$(3p - 5)(p - 2) =$
$-(x + 2)(x + 5) =$	$2(x + 4)(x + 9) =$
$(2x + 3)(4x^2 - 5x + 1) =$	

4. Factor fully:

$10x - 4 =$	$x^7 + x^4 =$
$-20x + 15 =$	$21x^2 + 14x^3 + 28x^5 =$
$16x^5 + 20x^3 =$	$24a^5b - 30a^2b^3 =$

Section 4: Factor a trinomial

5. Factor fully.

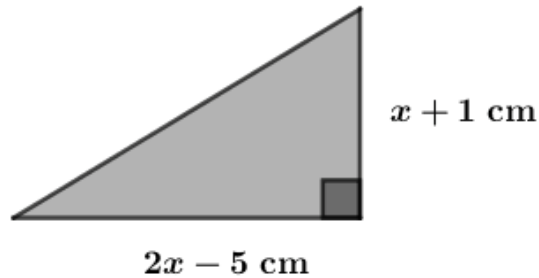
$x^2 + 3x + 2 =$	$x^2 + 6x + 9 =$
$x^2 + 12x - 1 =$	$x^2 - 11x + 10 =$
$x^2 - 5x - 14 =$	$x^2 - 49 =$

6. Complete the blanks:

$$(x + \quad)^2 = (x + \quad)(x + \quad) = x^2 + 10x + \quad .$$

Section 5: Mixed problems

10. A right-angled triangle has shorter sides measuring $(x + 1)$ cm and $(2x - 5)$ cm.



What mathematical properties do you notice about the diagram? What information has been provided? What information has not been provided?

Calculate the length of all three sides when $x = 3$.

For what value of x do the two shorter sides have equal length?

Using the Pythagorean Theorem, we can calculate that

$$c^2 = 5x^2 - 18x + 26$$

Write down the calculations required to arrive at this result.

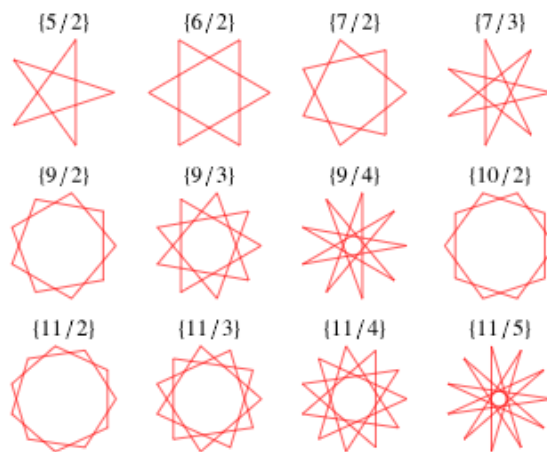
Use the formula for c^2 to find the length of the side labelled c when $x = 3$. Show your work.

Explain why the value of x in this problem must be greater than 2.5.

Knowledge: Reflect on these statements *without* referring to notes/a friend or other sources of knowledge. Circle 'yes' or 'no' or indicate a level of confidence:



I can explain what is meant by GCF	<input type="text" value="no"/> <input type="text" value="yes"/>
I can explain what is meant by LCM	<input type="text" value="no"/> <input type="text" value="yes"/>
I can list all the prime numbers up to 50	<input type="text" value="no"/> <input type="text" value="yes"/>
I can list at least 5 exponent laws	<input type="text" value="no"/> <input type="text" value="yes"/>
I can explain the word 'factor' in the context of an integer	<input type="text" value="no"/> <input type="text" value="yes"/>
I can explain the word 'factor' in the context of an algebraic expression	<input type="text" value="no"/> <input type="text" value="yes"/>
I can describe a polynomial using degree, number of variables and number of terms	<input type="text" value="no"/> <input type="text" value="yes"/>
I can identify a polynomial expression as either factored or written as a sum or neither	<input type="text" value="no"/> <input type="text" value="yes"/>
I can identify if a trinomial is a perfect or not perfect square	<input type="text" value="no"/> <input type="text" value="yes"/>
I can explain why some trinomials factor with integers and some don't	<input type="text" value="no"/> <input type="text" value="yes"/>
I can explain why a negative number raised to an exponent is sometimes negative and sometimes positive.	<input type="text" value="no"/> <input type="text" value="yes"/>
I can explain the difference between $-3x^2$ and $(-3x)^2$	<input type="text" value="no"/> <input type="text" value="yes"/>



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