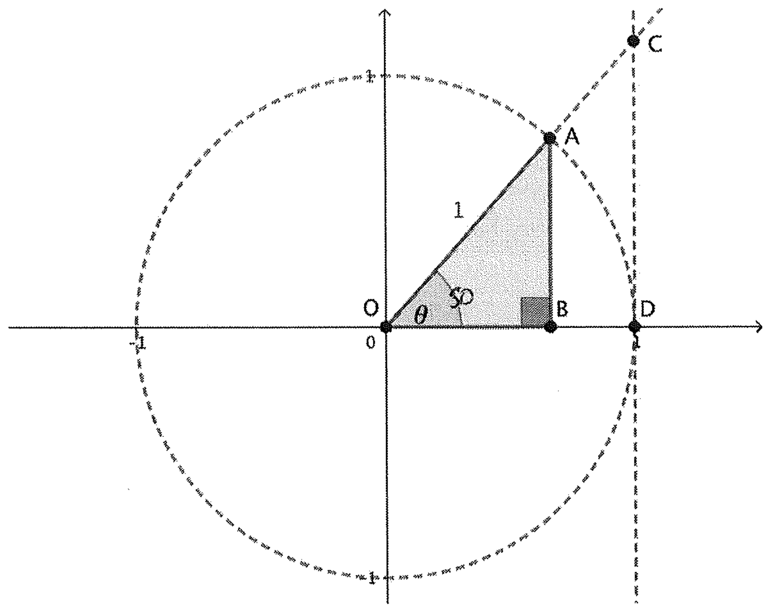
Grade 10 FMP Trigonometry Review Fall 2024 NAME: Solutions

Part 1: Theory

On the diagram below, AB is the side of the smaller triangle opposite the angle theta, OB is the side adjacent to the angle theta, OA has length 1 unit and DC is a length on the tangent of the circle.



(i) Let $\theta = 50°$
Circle all the true statements. Cross out any false statements.

$\sin(50°) = AB$	$\cancel{\cos(50°) = CD}$	$\tan(50°) = CD$	$\sin(40°) = OB$
$OA = 1$	$\cos(40°) = AB$	$\tan(50°) = \frac{AB}{OB}$	$\cancel{\sin(50°) = CD}$

(ii) Let the angle theta (θ) vary between 0° and 90°, that is, $0 \leq \theta \leq 90$. What is

(a) The minimum value of $\sin\theta$? At what angle? $\sin\theta = 1$ when $\theta = 90°$

(b) The maximum value of $\sin\theta$? At what angle? $\sin\theta = 0$ when $\theta = 0$

(iii) As the angle θ *increases* from 0 to 90°, one trig value *decreases*. Which one - $\sin\theta$ $\cos\theta$ or $\tan\theta$?

(iv) Without using a calculator, put these values in order of size. Explain your reason.
sin 23 °; cos 23°; tan 23°

Smallest: sin 23	Middle: tan 23	Largest: cos 23

Reason: ~~At 23° cosine~~ For angles less than 45°, cosine is bigger than sine. 23° is a small angle, so cosine is much larger. tan ~~23~~ is bigger than sine, but not much bigger at 23°.

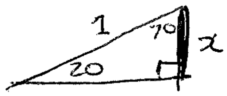
(v) Without using a calculator, put these values in order of size. Explain your reason.

cos 23°; cos 45°; cos 59°

Smallest: cos 59	Middle: cos 45	Largest: cos 23
Reason: cosine decreases as the angle increases.		

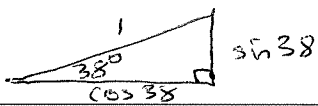
(vi) Without using a calculator, determine which two of the following five values are equal. Explain how you know.

tan 45°; sin 45°; cos 70°; cos 20°; sin 20°



because 20 + 70 = 90, they fit in the same right angled triangle.
cos 70 = x, sin 20 = x so sin 20 = cos 70

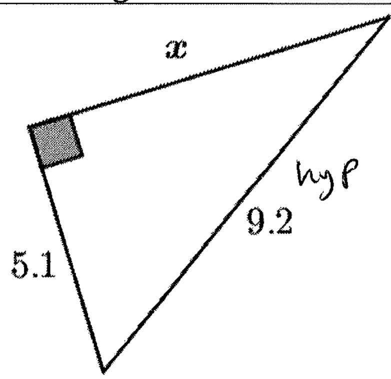
(vii)
Without using a calculator, explain how it is true that $(\sin(38°))^2 + (\cos(38°))^2 = 1$



They are three sides of a right angled triangle. (Pythagorean theorem)

Part 2: Two known values

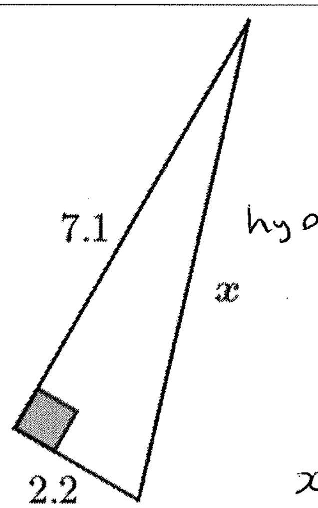
(i) Label the hypotenuse 'hyp'. Calculate the side labelled x. Check your answer by estimating.



$x^2 = 9.2^2 - 5.1^2$

$x = \sqrt{9.2^2 - 5.1^2}$

$= 7.7$

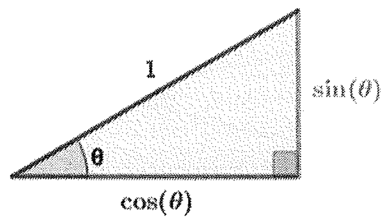


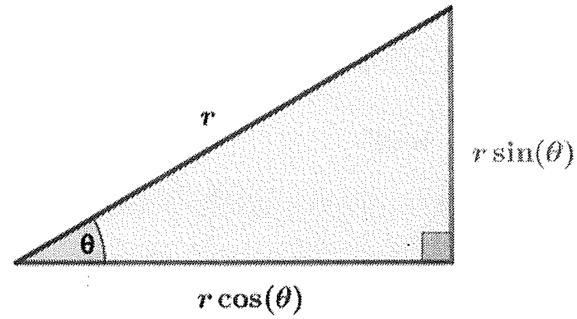
$x^2 = 7.1^2 + 2.2^2$

$x = \sqrt{7.1^2 + 2.2^2}$

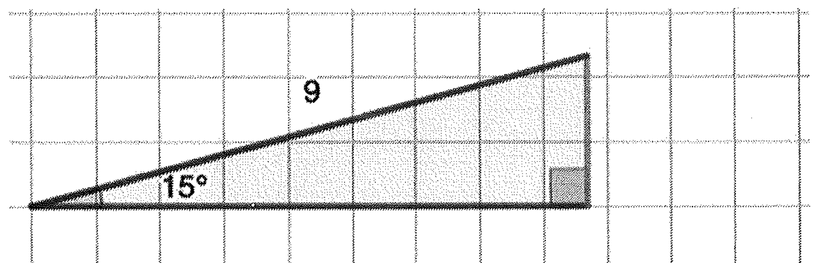
$= 7.4$

(ii) Using the enlargement principle.





Given that $\cos(15°) = 0.966$ and $\sin(15°) = 0.259$, calculate the adjacent and opposite sides of this triangle. Show your working.



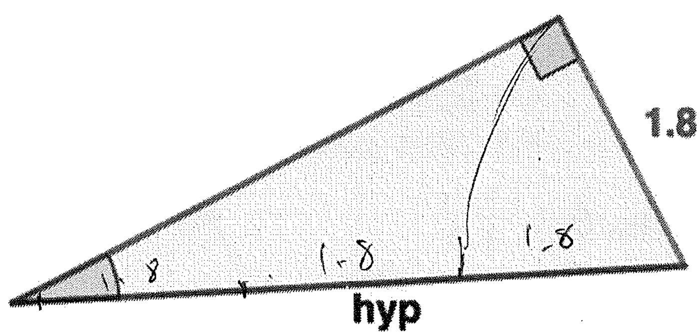
adjacent = 9×0.966 $= 8.694$

opposite = 9×0.259 $= 2.331$

or equivalent working.

(iii) Estimation

One shorter leg of this triangle measures 1.8 units. The triangle is drawn to scale. *Estimate* the length of the hypotenuse. Give your reasoning.

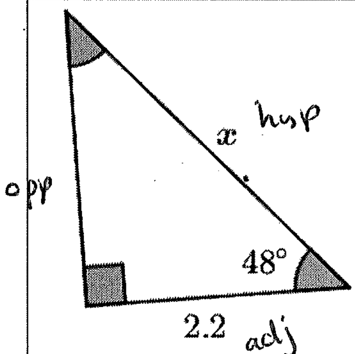


Approximately $3 \times 1.8 = 5.4$ ← a bit high it turns out.

by measuring + calculating, $\frac{1.8}{37} = \frac{x}{90}$; $x = 4.4$

1 d.p.

(iv) Calculate the side or the angle marked. Show all lines of work. Triangles are drawn to scale – use estimation to check your answer.

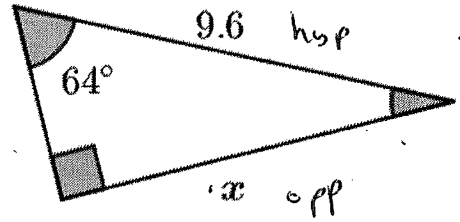


$\cos 48 = \frac{2.2}{x}$

$x = \frac{2.2}{\cos 48}$

$= \cancel{3.29}$

$= 3.3$



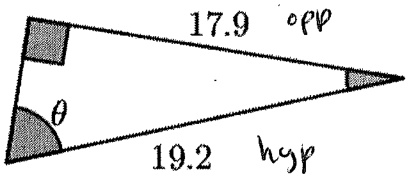
$\cancel{\sin 64 = \frac{9.6}{x}}$

$\cancel{x =}$

$\sin 64 = \frac{x}{9.6}$

$x = 9.6 \times \sin 64$

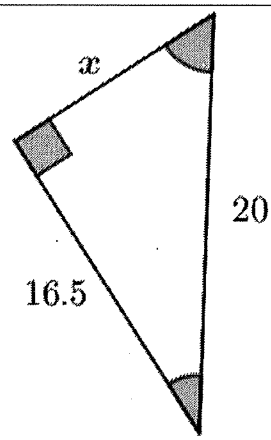
$= 8.6$



$\sin \theta = \frac{17.9}{19.2}$

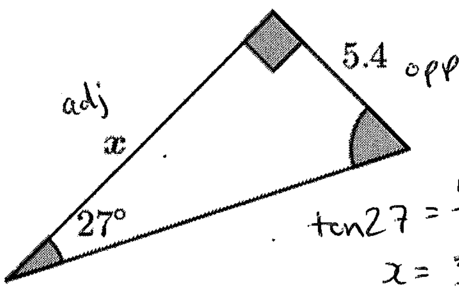
$\theta = \sin^{-1}\left(\frac{17.9}{19.2}\right)$

$\theta = 68.8°$



$x^2 = 20^2 - 16.5^2$

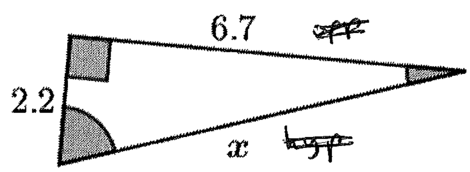
$x = 11.3$



$\tan 27 = \frac{5.4}{x}$

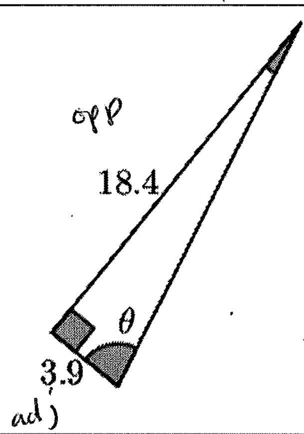
$x = \frac{5.4}{\tan 27}$

$x = 10.6$



$x^2 = 6.7^2 + 2.2^2$

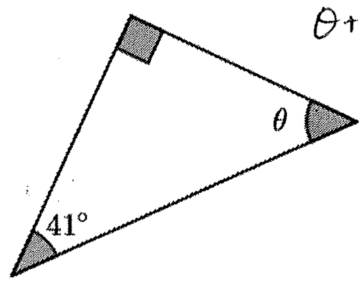
$= 7.1$



$\tan \theta = \frac{18.4}{3.9}$

$\theta = \tan^{-1}\left(\frac{18.4}{3.9}\right)$

$= 78°$



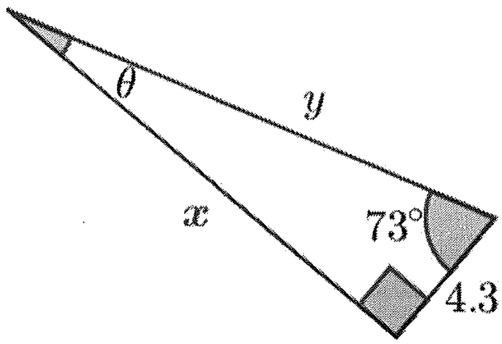
$\theta + 41 = 90$

$\theta = 49°$

Part 3: Problem Solving

#1

Solve this triangle. Find all unknown sides and angles.



$\tan 73 = \frac{x}{4.3}$

$\boxed{x = 4.3\tan 73 = 14.1}$

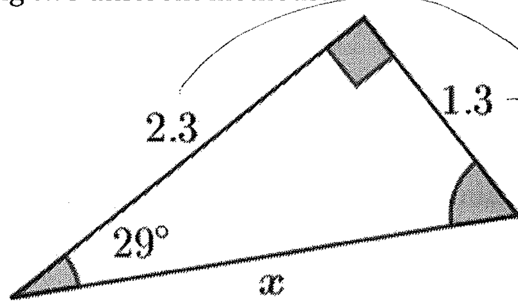
$\cos 73 = \frac{4.3}{y}$

$\boxed{y = \frac{4.3}{\cos 73} = 14.7}$

$\boxed{\theta = 90 - 73 = 17°}$

 # 2

Calculate the value of x using two different methods:



Answers in the 2nd decimal place vary as 2.3 and 1.3 are rounded lengths.

Method 1

$x^2 = 2.3^2 + 1.3^2$

$x = \sqrt{2.3^2 + 1.3^2}$

$= 2.64...$

Method 2

$\sin 29 = \frac{1.3}{x}$

$x = \frac{1.3}{\sin 29}$

$x = 2.68...$

Method 3

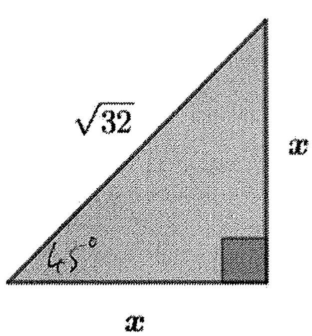
$\cos 29 = \frac{2.3}{x}$

$x = \frac{2.3}{\cos 29}$

$x = 2.63...$

 # 3

Find the value of x, given that



By Pythagoras, $x^2 + x^2 = (\sqrt{32})^2$

$2x^2 = 32$

$x^2 = 16$

$x = 4$

Or, angles are 45° as triangle is isosceles.

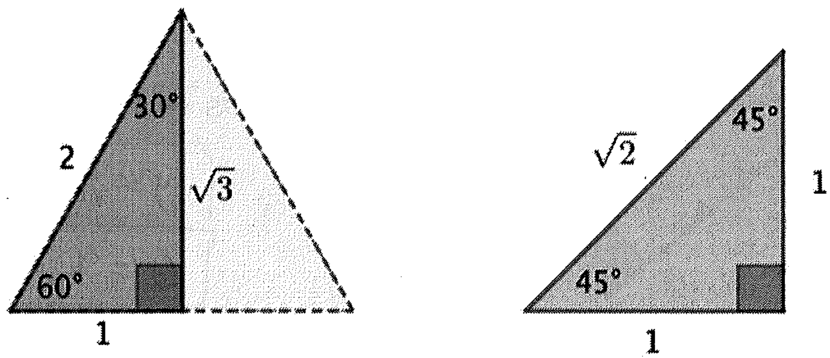
$\cos 45 = \frac{x}{\sqrt{32}}$

$x = \sqrt{32}\cos 45 = 4$

#4:

The following triangles are known as 'exact value triangles'. They provide a way to calculate the sine, tangent, cosine of 60°; 30° and 45° without using a table of values or a calculator.

The first is an equilateral triangle cut in half; the second is an isosceles, right-angled triangle.



Because $\sin\theta = \frac{opp}{hyp}$, $\sin 60° = \frac{\sqrt{3}}{2}$

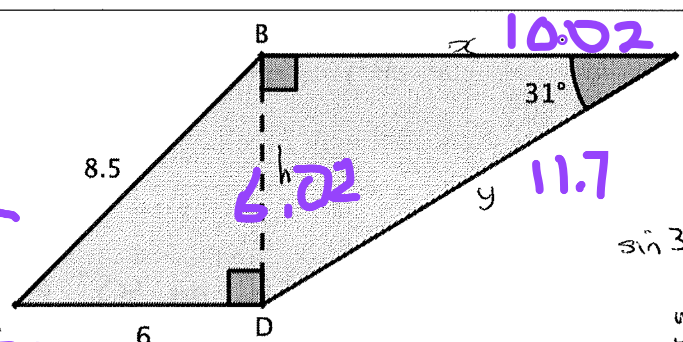
Use your calculator to verify that $\frac{\sqrt{3}}{2}$ is *exactly* equal to sin 60.

Use the values on the triangles above to write fractions for each trig value:

$\sin 30° = \frac{opp}{hyp} = \frac{1}{2}$	$\sin 60° = \frac{\text{opp}}{\text{hyp}} = \frac{\sqrt{3}}{2}$	$\sin 45° = \frac{opp}{hyp} = \frac{1}{\sqrt{2}}$
$\cos 30° = \frac{adj}{hyp} = \frac{\sqrt{3}}{2}$	$\cos 60° = \frac{adj}{hyp} = \frac{1}{2}$	$\cos 45° = \frac{1}{\sqrt{2}}$
$\tan 30° = \frac{\text{opp}}{\text{adj}} = \frac{1}{\sqrt{3}}$	$\tan 60° = \frac{opp}{adj} = \frac{\sqrt{3}}{1} = \sqrt{3}$	$\tan 45° = \frac{opp}{adj} = \frac{1}{1} = 1$

 # 5:

Calculate the perimeter of quadrilateral ABCD.



$h^2 = 8.5^2 - 6^2 =$

$h = \sqrt{\ldots} = 6.02$

$\tan 31 = \frac{h}{x}$

$x = \frac{h}{\tan 31} = 10.02$

$= \cancel{\frac{\sqrt{\ldots}}{\tan 31}}$

$= \cancel{17.34\ldots}$

$\sin 31 = \frac{h}{y}$

$y = \frac{h}{\sin 31} = 11.7$

$= \cancel{\frac{\sqrt{\ldots}}{\sin 31}}$

$= \cancel{20.20137}$

$P = 8.5 + 6 + \cancel{12.3 + 20.2}$ $10 + 11.7$

$= \cancel{52}$ 36.2