PreCalculus Quadratic Functions and Equations Assignment #2 10%

1.Write each of these quadratic functions in the form $f(x) = a(x-h)^2 + k$ (completed square form)

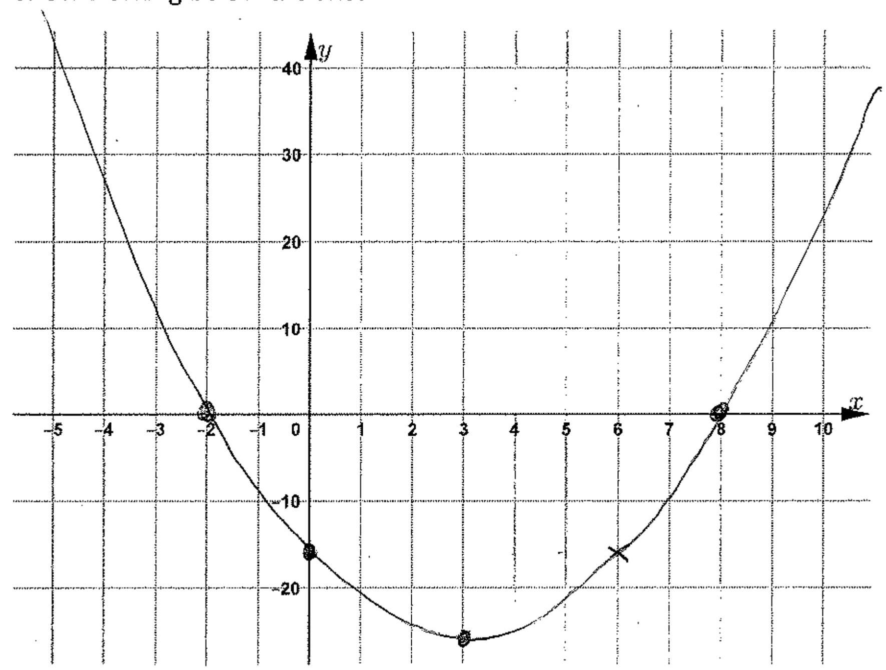
$f(x) = x^2 + 8x - 3$ $= (x+4)^2 - 16 - 3$ $= (x+4)^2 - 19$	$f(x) = x^2 - 5x + 4$ $= \left(x - \frac{5}{2}\right)^2 - \frac{25}{4} + 4$ $= \left(x - \frac{5}{2}\right)^2 - \frac{25}{4} + \frac{16}{4}$ $= \left(x - \frac{5}{2}\right)^2 - \frac{9}{4}$	$f(x) = 2x^2 + 6x + 5$ $= 2\left(x^2 + 3x + \frac{5}{2}\right)$ $= 2\left(\left(x + \frac{3}{2}\right)^2 - \frac{9}{4} + \frac{5}{2}\right)$ $= 2\left(x + \frac{3}{2}\right)^2 - \frac{9}{2} + 5$ $= 2\left(x + \frac{3}{2}\right)^2 - \frac{9}{2} + \frac{10}{2}$ $= 2\left(x + \frac{3}{2}\right)^2 + \frac{1}{2}$
$f(x) = x^2 - 2x + 7$ $= (x-1)^2 - 1 + 7$ $= (x-1)^2 + 6$	$f(x) = x^2 + 7x - 2$ $= \left(x + \frac{7}{2}\right)^2 - \frac{49}{4} - 2$ $= \left(x + \frac{7}{2}\right)^2 - \frac{49}{4} - \frac{8}{4}$ $= \left(x + \frac{7}{2}\right)^2 - \frac{57}{4}$	$f(x) = 3x^2 - 5x - 4$ $= 3\left(x^2 - \frac{5}{3}x - \frac{4}{3}\right)$ $= 3\left(\left(x - \frac{5}{6}\right)^2 - \frac{25}{36} - \frac{4}{3}\right)$ $= 3\left(x - \frac{5}{6}\right)^2 - \frac{25}{12} - 4$ $= 3\left(x - \frac{5}{6}\right)^2 - \frac{25}{12} - \frac{48}{12}$ $= 3\left(x - \frac{5}{6}\right)^2 - \frac{73}{12}$

2.Write each of these quadratic functions in factored form:

$f(x) = x^2 - 13x + 30$ $= (x-10)(x-3)$	$f(x) = 3x^2 + x - 2$ $= (3x-2)(x+1)$
$f(x) = x^2 + 18x + 77$ $= (x+11)(x+7)$	$f(x) = 5x^2 - 13x - 6$ $= (5x+2)(x-3)$
$f(x) = x^2 - 8x$ $= x(x-8)$	$f(x) = 4x^2 - 2x - 2$ $= 2(2x^2 - x - 1)$ $= 2(2x+1)(x-1)$
$f(x) = 4x^2 - 16$ $= 4(x^2-4)$ $= 4(x-2)(x+2)$	$f(x) = 10x^2 + 21x + 2$ $= (x+2)(10x+1)$

3. Draw the graph $y = f(x)$ where $f(x) = x^2 - 6x - 16$. Calculate, plot and label the y-intercept, the x-intercepts and the vertex.

Show working below the axes.



y int; $x = 0$ $\quad y = 0^2 - 6(0) - 16 = -16$

Complete square: $(x-3)^2 - 9 - 16$

$= (x-3)^2 - 25$ $\quad$ vertex $(3, -25)$

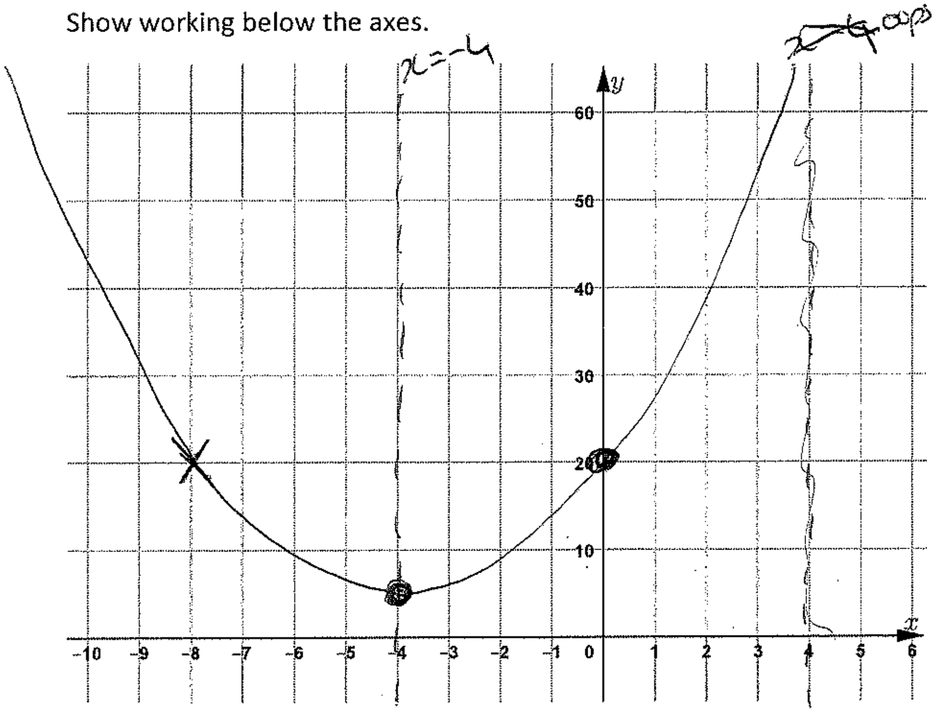
x int, $y = 0$ $\quad 0 = x^2 - 6x - 16$

$0 = (x+2)(x-8)$

$x = -2$ or $x = 8$

4. Draw the graph $y = f(x)$ where $f(x) = x^2 + 8x + 20$. Calculate, plot and label the y-intercept, the vertex, and the point which is the reflection of the y-intercept over the line of symmetry of the parabola.

Show working below the axes.



y int, $x=0$ $\quad f(0)=0^2+8(0)+20=20$

$(0,20)$

Vertex: $x = \frac{-b}{2a} = \frac{-8}{2(1)} = -4$.

$y = f(-4) = (-4)^2 + 8(-4) + 20$

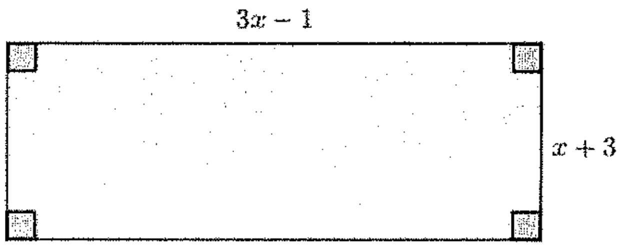
$= 16 - 32 + 20$

$= -16 + 20 \quad = 4$

5. Solve the following equations for x. If solutions are irrational, give them both in exact form and correct to 2 decimal places.

(a) $x^2 = 36$ $x = \pm 6$	(b) $(x-5)(2x+3) = 0$ $x-5=0 \quad 2x+3=0$ $x=5 \quad 2x=-3$ $x=-3/2$
(c) $(x+6)^2 - 3 = 0$ $(x+6)^2 = 3$ $x+6 = \pm\sqrt{3}$ $x = -6 \pm \sqrt{3}$ $x_1 = -6-\sqrt{3}, \quad x_2 = -6+\sqrt{3}$	(d) $x^2 + 7x - 8 = 0$ $(x+8)(x-1) = 0$ $x+8=0 \quad x-1=0$ $\boxed{x=-8} \quad \boxed{x=1}$
(e) $3x^2 - 16 = 0$ $3x^2 = 16$ $x^2 = \frac{16}{3}$ $x = \pm\sqrt{\frac{16}{3}} =$ $x_1 = -\frac{4}{\sqrt{3}} \quad x_2 = 4/\sqrt{3}$	(f) $2x^2 + 7x + 3 = 2x + 4$ $2x^2+5x-1=0$ $x = \frac{-5 \pm \sqrt{25+8}}{4}$ $x_1 = \frac{-5-\sqrt{33}}{4} \quad x_2 = \frac{-5+\sqrt{33}}{4}$ $-2.69 \quad 0.19$
(g) $\frac{2x}{x-9} = \frac{x+4}{2}$ $4x = (x+4)(x-9)$ $4x = x^2-5x-36$ $0 = x^2-9x-36$ $0 = (x-12)(x+3)$ $x-12=0$ or $x+3=0$ $x=12$ or $x=-3$	

6. The area of the rectangle below is 312 cm^2. Calculate the lengths of each side.



$\boxed{x > \frac{1}{3} \text{ for positive lengths}}$

Area = length × width

$= (3x-1)(x+3)$

$= 3x^2 + 8x - 3$

Now Area = 312, so

$312 = 3x^2 + 8x - 3$

$0 = 3x^2 + 8x - 315$

$0 = (3x+35)(x-9)$

$3x + 35 = 0$

$3x = -35$

$x = -\frac{35}{3}$

reject because this leads to negative side lengths.

$x - 9 = 0$

$\boxed{x = 9}$

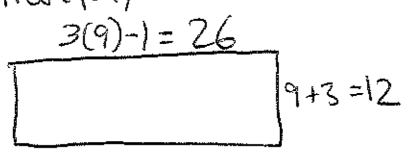
~~309~~ 315

~~3 103~~ 45 7

$\boxed{3}$ 9 5

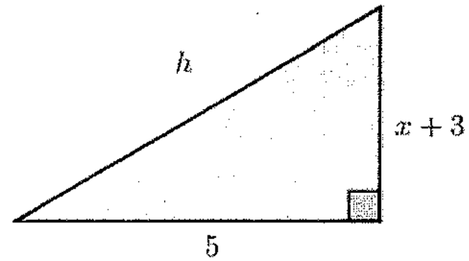
27 35

Therefore, sides are



$26 \times 12 = 312$ ✓

7. A right angled triangle has shorter sides with lengths 5 and $(x+3)$ as shown in the figure.



$x > -3$

(a) Write the length of the hypotenuse h as a function of x.

(b) Find the value of x for which the hypotenuse $h = 10$. Write your answer in simplified, exact form and as a decimal correct to 2 decimal places.

By Pythagoras' theorem,

$h^2 = (x+3)^2 + 5^2$

$= (x+3)(x+3) + 25$

$= x^2 + 6x + 9 + 25$

$= x^2 + 6x + 34$

Therefore, $h(x) = \sqrt{x^2+6x+34}$

(b) Given that $h = 10$, we have

$10 = \sqrt{x^2+6x+34}$

$\Rightarrow \quad 100 = x^2 + 6x + 34$

$0 = x^2 + 6x - 66$

$0 = (x+3)^2 - 75$

$75 = (x+3)^2$

$\pm\sqrt{75} = x + 3$

$x = \pm\sqrt{75} - 3$

Now

$\sqrt{75} = \sqrt{25 \times 3}$

$= 5\sqrt{3}$

$x_1 = -3 - 5\sqrt{3}$ reject

$x_2 = -3 + 5\sqrt{3}$

As $x > -3$

$\boxed{x = -3 + 5\sqrt{3}}$

$(x = 5.66)$