PC 11 Course Review

FINANCE

F1. $55 \times 1.12 = 61.6$ \$61.60 after tax.

F2. $(12995 + 299) \times 1.12 =$ ~~[illegible]~~ \$14889.28

F3. $550000 = 200000 + 350000$

$$\text{TAX} = 0.01 \times 200000 + 0.02 \times 350000$$

$$= 2000 + 7000 = \underline{\$9000} \text{ tax}$$

F4. $2300000 = 200000 + 1800000 + 300000$

$$\text{TAX} = 0.01 \times 200000 + 0.02 \times 1800000 + 0.03 \times 300000$$

$$= 2000 + 36000 + 9000$$

$$= \$47000$$

F5. $PV = 9100$, $t = 5$, $r = 0.056$, $n = 365$

$$FV = 9100\left(1 + \frac{0.056}{365}\right)^{5 \times 365} = \$12040.22$$

F6. $PV = 7900$ $t = 10$, $r = 0.014$, $n = 52$

$$FV = 7900\left(1 + \frac{0.014}{52}\right)^{10 \times 52} = 9086.99$$

$$\text{Interest} = FV - PV = 9086.99 - 7900 = \$1186.99$$

F7. ~~PV=~~ → next

F7. $RoR = \frac{FV - PV}{PV} \times 100$

$PV = 6000; \quad t = 25; \quad r = 0.035, \quad n = 12$

$FV = 6000\left(1 + \frac{0.035}{12}\right)^{12 \times 25} = 14374.93$

$RoR = \frac{14374.93 - 6000}{6000} \times 100 = 140\%$ (nearest %)

F8. $PV = FV \div \left(1 + \frac{r}{n}\right)^{n \times t}$

$= 56783.82 \div \left(1 + \frac{0.067}{52}\right)^{40 \times 52}$

$= \$3900$

F9. TVM fields

PV	13000
Payments	-389.04
FV	0
Annual Rate(%)	4.9
Periods	36
Compounding	monthly

Payments are \$~~387.29~~ 389.04 / month for 3 years.

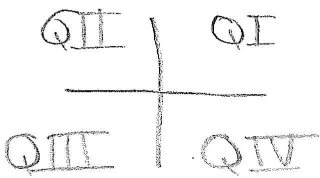
F10.

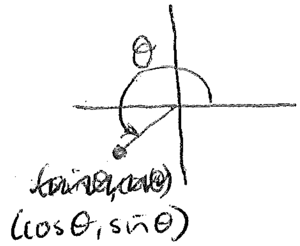
PV	-2000
Payments	-100
FV	43723.37
Annual Rate(%)	4.5
Periods	240
Compounding	monthly

The future value of this investment will be \$43723.37, provided nothing changes.

TRIGONOMETRY

T1.





(a) $\sin\theta$ is negative (y co-ord)

(b) $\cos\theta$ is negative (x co-ord)

(c) $\tan\theta$ is positive $\left(\frac{y}{x}\right)$

T2.

$\sin^2\theta + \cos^2\theta = 1$ ← Pythagorean Identity

$(\sin\theta)^2 + (\cos\theta)^2 = 1$

$(\sin\theta)^2 + \left(\frac{2}{5}\right)^2 = 1$

$(\sin\theta)^2 + \frac{4}{25} = 1$

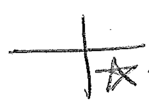
$(\sin\theta)^2 = 1 - \frac{4}{25}$

$(\sin\theta)^2 = \frac{25}{25} - \frac{4}{25}$

$(\sin\theta)^2 = \frac{21}{25}$

$\sin\theta = \pm\sqrt{\frac{21}{25}} = \frac{\pm\sqrt{21}}{\cancel{2}5}$

Now θ lies in quadrant IV

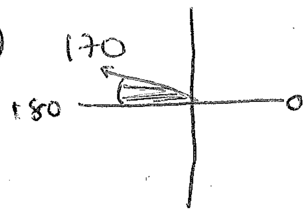


← here.

So $\sin\theta$ is negative.

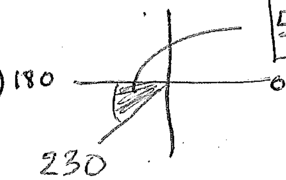
Therefore, $\sin\theta = \frac{-\sqrt{21}}{5}$

T3 (a)



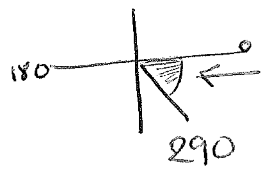
$\boxed{10^\circ}$

(b)



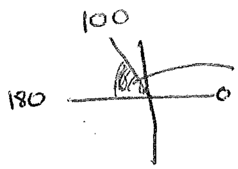
$\boxed{50^\circ}$

(c)



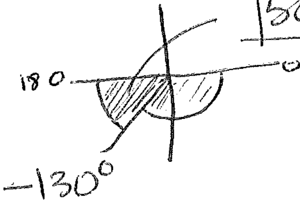
$\boxed{70^\circ}$

(d)



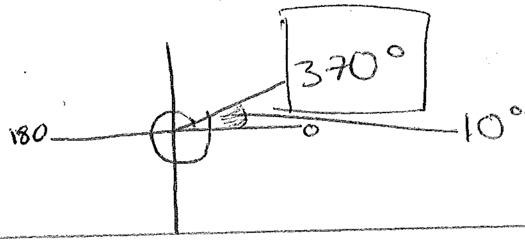
$\boxed{80^\circ}$

(e)



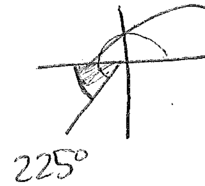
$\boxed{50^\circ}$

(f)



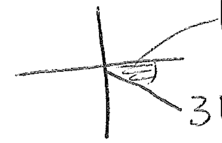
$\boxed{370^\circ}$

(g)



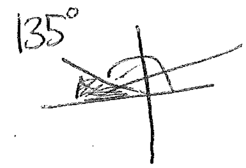
$\boxed{45^\circ}$

(h)



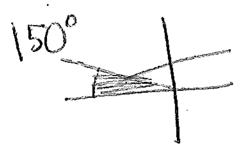
$\boxed{45^\circ}$

(i)



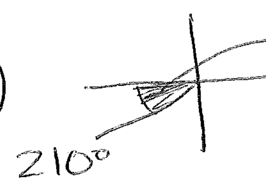
$\boxed{45^\circ}$

(j)



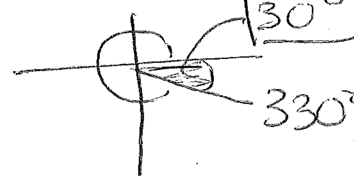
$\boxed{30^\circ}$

(k)



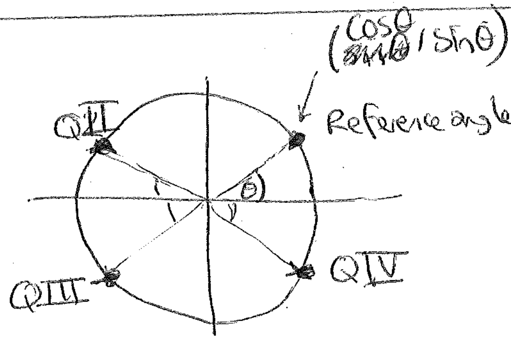
$\boxed{30^\circ}$

(l)

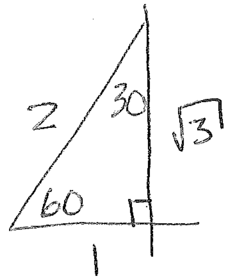


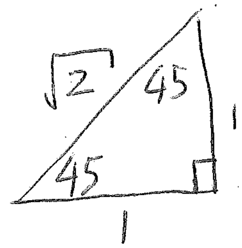
$\boxed{30^\circ}$

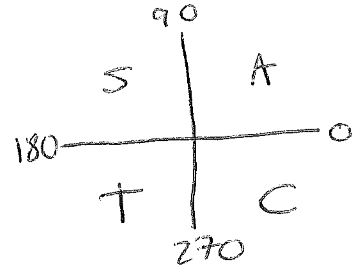
Reference angles?



Due to the symmetry of the circle, these 4 locations have the same (x, y) coordinates ~~only~~ as the reference angle, only the sign $\pm$ changes.



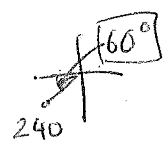




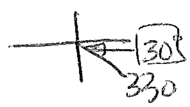
T4 (a) $\cos 30° = \frac{\sqrt{3}}{2}$

(b) $\tan 60° = \frac{\sqrt{3}}{1} = \sqrt{3}$

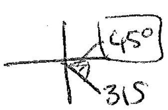
(c) $\sin 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ (rationalized denominator)



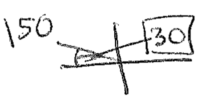
(d) $\sin 240° = -\sin 60° = -\frac{\sqrt{3}}{2}$



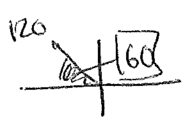
(e) $\cos 330° = \cos 30° = \frac{\sqrt{3}}{2}$



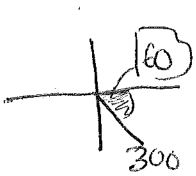
(f) $\tan 315° = -\tan 45° = -1$



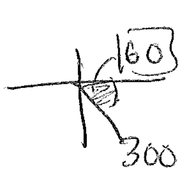
(g) $\sin 150° = \sin 30° = \frac{1}{2}$



(h) $\tan 120° = -\tan 60° = -\sqrt{3}$



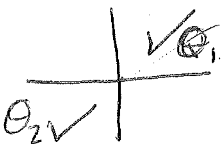
(i) $\cos 300° = \cos 60° = \frac{1}{2}$



(j) $\tan 300° = -\tan 60 = -\sqrt{3}$

T5.

(a) $\tan\theta = 0{,}9$



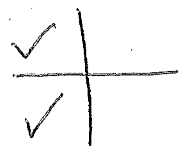
Answers in QI and QIII

$\theta_1 = \tan^{-1}(0{,}9) = 41{,}987\ldots$

$\theta_2 = 180 + 41{,}987\ldots = 221{,}987\ldots$

Solution Set $\theta \in \{42^\circ; 222^\circ\}$ (1 d.p)

(b) $\cos\theta = -0{,}7$



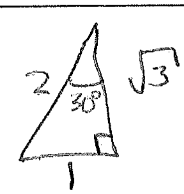
Answers in QII and QIII

$\theta_1 = \cos^{-1}(-0{,}7) = 134{,}4^\circ$

$\theta_2 = 360 - 134{,}4 = 225{,}6$

Solution set $\theta \in \{134{,}4^\circ, 225{,}6^\circ\}$

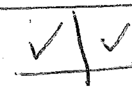
(c) Answers in QII and QIII.



reference angle from triangle or calculator,

Solution set $\theta \in \{150^\circ, 210^\circ\}$

(d) Answers in QI and QII (sine positive)

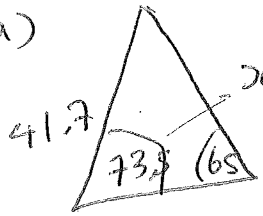


$\theta_1 = \sin^{-1}(0{,}7) = 44{,}4$

$\theta_2 = 180 - 44{,}4 = 135{,}6$

Solution Set $\theta \in \{44{,}4^\circ, 135{,}6^\circ\}$

T6 (a)



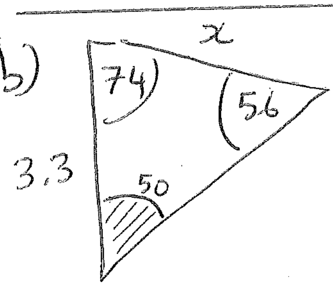
Use sine rule,

$$\frac{x}{\sin 73.5} = \frac{41.7}{\sin 65}$$

$$x = \frac{41.7 \times \sin 73.5}{\sin 65} = \underline{\underline{44.12}}$$

(2 d.p)

(b)



Calculate 3rd angle: $180 - 74 - 56 = 50$

Use sine rule

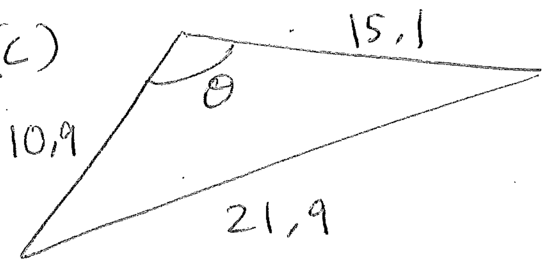
$$\frac{x}{\sin 50} = \frac{3.3}{\sin 56}$$

$$x = 3.3 \times \sin 50 \div \sin 56$$

$$= \underline{\underline{3.05}}$$

(2 d.p.)

(c)



Notice that θ is obtuse, that is, is between $90°$ and $180°$.

Use cosine law

$$\cos\theta = \frac{10.9^2 + 15.1^2 - 21.9^2}{2 \times 10.9 \times 15.1}$$

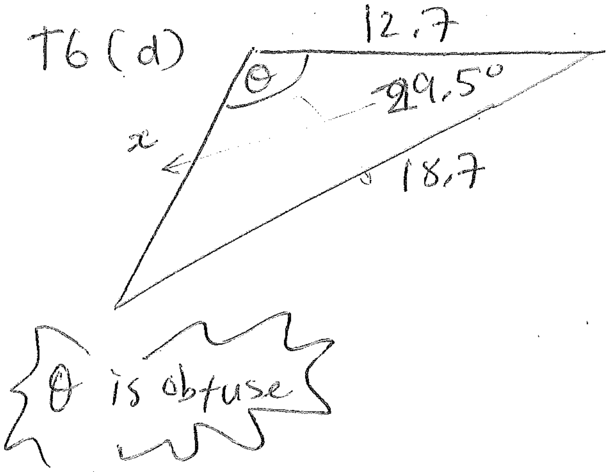
$$= \frac{-132.79}{329.18}$$

$$\theta = \cos^{-1}(\text{ANS})$$

$$= \underline{\underline{114°}}$$

(nearest degree)

T6 (d)



double work - use cosine rule first

$x^2 = 18.7^2 + 12.7^2 - 2(18.7)(12.7)\cos 29.5$

$x = \sqrt{ANS}$

$= 9.87818\ldots$

$\frac{\sin\theta}{18.7} = \frac{\sin 29.5}{9.87818}$

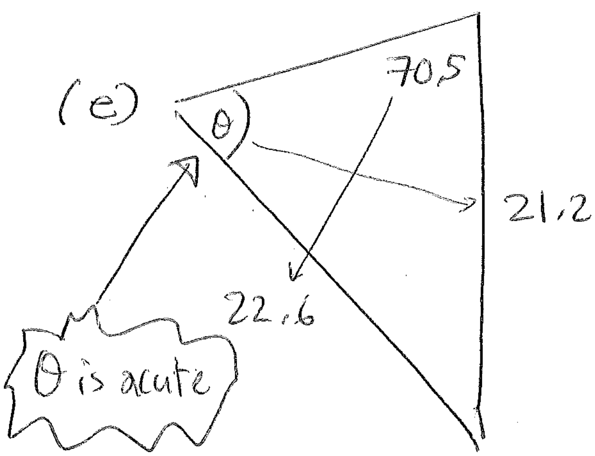
$\sin\theta = 18.7 \times \sin 29.5 \div 9.87818\ldots$

~~$\theta = \sin^{-1}(ANS)$~~ θ is obtuse

$\theta = 180 - \sin^{-1}(ANS)$

$= \underline{111.2°}$ (1d.p)

(e)



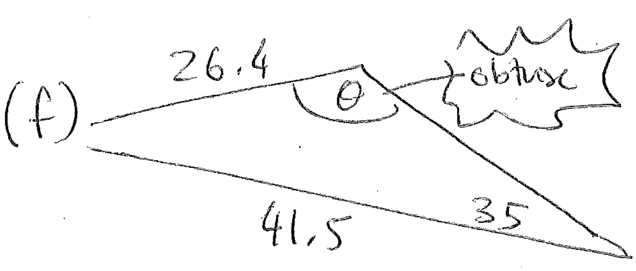
$\frac{\sin\theta}{21.2} = \frac{\sin 70.5}{22.6}$

$\sin\theta = \frac{21.2 \times \sin 70.5}{22.6}$

$\theta = \sin^{-1}(ANS)$

$\theta = 62.2°$ (1dp)

(f)



$\frac{\sin\theta}{41.5} = \frac{\sin 35}{26.4}$

$\sin\theta = 41.5 \times \sin 35 \div 26.4$

$\theta = 180 - \sin^{-1}(ANS)$

$= \underline{116°}$ (nearest degree)

QUADRATIC FUNCTIONS & EQUATIONS

Q1

(a) $5x^2+7x-6 = (x+2)(5x-3)$

Factor $x^2+7x-30$
$= (x+10)(x-3)$
$= (x+5\cdot 2)(x-3)$

(b) $4x^2-11x+6 = (x-2)(4x-3)$

Factor $x^2-11x+24$
$= (x-8)(x-3)$
$= (x-4\cdot 2)(x-3)$

(c) $2x^2+8x+8 = 2(x^2+4x+4)$
$= 2(x+2)(x+2)$
$= 2(x+2)^2$

common factor 2

(d) $-x^2+x+12 = -(x^2-x-12)$
$= -(x-4)(x+3)$

factor -1

(e) $3x^2-6x-9 = 3(x^2-2x-3)$
$= 3(x-3)(x+1)$

factor 3

(f) over

Q1 (f) $2x^2 - 2 = 2(x^2 - 1)$

difference of two squares $= 2(x-1)(x+1)$

(g) $x^2 - 100 = (x-10)(x+10)$

Q2. $y = x^2 + 4x - 60$

x intercepts occur when $y = 0$

solve

$0 = x^2 + 4x - 60$

(a) $a = 1, \quad b = 4, \quad c = -60$

$$x = \frac{-4 \pm \sqrt{16 + 240}}{2(1)}$$

$$= \frac{-4 \pm \sqrt{256}}{2}$$

$x_1 = \frac{-4-16}{2} \qquad x_2 = \frac{-4+16}{2}$

$x_1 = -10 \qquad x_2 = 6$

(b) $0 = x^2 + 4x - 60$

$0 = (x+2)^2 - 4 - 60$

$0 = (x+2)^2 - 64$

$64 = (x+2)^2$

$\pm\sqrt{64} = x + 2$

$x = -2 \pm 8$

$x_1 = -2-8 \qquad x_1 = -2+8$

$= -10 \qquad = 6$

(c) $0 = x^2 + 4x + 60$

$0 = (x+10)(x-6)$

$x + 10 = 0$ or $x - 6 = 0$

$x = -10$ or $x = 6$.

Q3 (a) $y = x^2 + 6x + 9$

$0 = x^2 + 6x + 9$

$0 = (x+3)(x+3)$

$x = -3$

(1 repeated root)

(b) $y = x^2 - 6x - 4$

$0 = x^2 - 6x - 4$

$\cancel{= (\quad)(\quad)}$

doesn't factor

$0 = x^2 - 6x - 4$

$0 = (x-3)^2 - 9 - 4$

$0 = (x-3)^2 - 13$

$13 = (x-3)^2$

$\pm\sqrt{13} = x - 3$

$x = 3 \pm \sqrt{13}$

$x_1 = -0.606$; $x_2 = 6.61$

(c) $0 = 5x^2 + 20x + 11$

$a = 5,\ b = 20,\ c = 11$

$x = \dfrac{-20 \pm \sqrt{400 - 220}}{10}$

$= \dfrac{-20 \pm \sqrt{180}}{10}$

$x_1 = -3.34$; $x_2 = -0.658$

exact form

$x_p = \dfrac{-20 \pm \sqrt{36 \times 5}}{10}$

$= \dfrac{-20 \pm 6\sqrt{5}}{10}$

$= \dfrac{-10 \pm 3\sqrt{5}}{5}$

$x_1 = \dfrac{-10 - 3\sqrt{5}}{5}$ $x_2 = \dfrac{-10 + 3\sqrt{5}}{5}$

(d) $y = -x^2 + 10x - 22$

doesn't factor.

$y = -(x^2 + 10x - 22)$

$y = -((x+5)^2 - 25 - 22)$

$0 = -((x+5)^2 - 47)$

$0 = -(x+5)^2 + 47$

$-47 = -(x+5)^2$

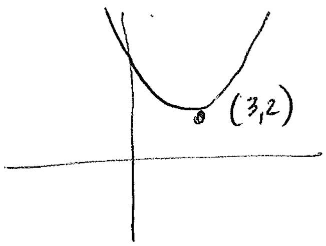
$47 = (x+5)^2$

$\pm\sqrt{47} = x + 5$

$x_1 = -5 - \sqrt{47}$ $x_2 = -5 + \sqrt{47}$

$= -11.9$, $x_2 = 1.86$

Q4



vertex above the x axis, upwards facing

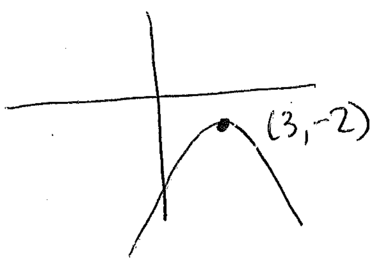
For example,

$$y = (x-3)^2 + 2$$

Change form to standard form

$$y = (x-3)(x-3) + 2$$
$$= x^2 - 6x + 9 + 2$$
$$= \underline{x^2 - 6x + 11}$$

or



vertex below the x axis, downwards facing.

For example

$$y = -(x-3)^2 - 2$$
$$= -(x-3)(x-3) - 2$$
$$= -(x^2 - 6x + 9) - 2$$
$$= -x^2 + 6x - 9 - 2$$
$$= \underline{\underline{-x^2 + 6x - 11}}$$

Q5. (a) $x^2 - 36 = 0$ or $x^2 = 36$

$x = \pm\sqrt{36}$

$(x-6)(x+6) = 0$

$x = \pm 6$

$x = 6$ or $x = -6.$

(b) $(x-5)(2x+9) = 0$

either $x - 5 = 0$ or $2x + 9 = 0$

$\Rightarrow$ $x = 5$ $\Rightarrow$ $2x = -9$

$\Rightarrow$ $x = \frac{-9}{2}$

Solution set $x \in \{-\frac{9}{2}, 5\}$

(c) $5x^2 + 26x = 6x - 11$

$5x^2 + 20x + 11 = 0$ this is Q3 (c)

$x_1 = -3.34$ or $x_2 = -0.658$

exact form $x_1 = \frac{-10 - 3\sqrt{5}}{5}$, $x_2 = \frac{-10 + 3\sqrt{5}}{5}$

(d) $x - 6 = \frac{4}{x}$ multiply both sides by x

$x^2 - 6x = 4$

$x^2 - 6x - 4 = 0$

$(x-3)^2 - 13 = 0$

$13 = (x-3)^2$

$\pm\sqrt{13} = x - 3$ add 3

$x = 3 \pm \sqrt{13}$

$x_1 = -0.61$, $x_2 = 6.61$

Q5 (e) $x^2 - 8x - 1 = 0$

$a=1 \quad b=-8 \quad c=-1$

$$x = \frac{\overset{8}{-b} \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{8 \pm \sqrt{64+4}}{2}$$

$$= \frac{8 \pm \sqrt{68}}{2}$$ → from here, you can calculate decimal

↓ or continue to leave in simplified, exact form.

$$= \frac{8 \pm \sqrt{4 \times 17}}{2}$$

$$= \frac{8 \pm 2\sqrt{17}}{2}$$

$$= 4 \pm \sqrt{17}$$

or

$$x^2 - 8x - 1 = 0$$
$$(x-4)^2 - 17 = 0$$
$$(x-4)^2 = 17$$
$$x - 4 = \pm\sqrt{17}$$
$$x = 4 \pm \sqrt{17}$$

$x_1 = -0.12$

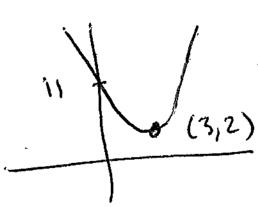
$x_2 = 8.12$

(f) $(x-4)^2 - 17 = 0$ is the same question as part e,

so $x_1 = -0.12$, $x_2 = 8.12$

Q6. To write the range we need to know Yvertex, and open up V or open down Λ.

eg



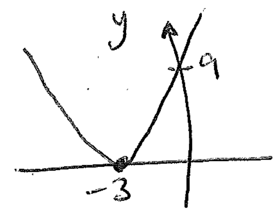
This parabola has range $y \geq 2$.

(All points on the parabola have a y coordinate greater than 2.)

Q6 (a) $y = x^2 + 6x + 9$

$= (x+3)^2 - 9 + 9$

$= (x+3)^2$ vertex $(-3, 0)$

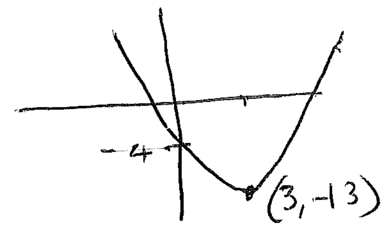


Range $y \geq 0$

(b) $y = x^2 - 6x - 4$

$= (x-3)^2 - 9 - 4$

$= (x-3)^2 - 13$ vertex $(3, -13)$



Range $y \geq -13$

(c) $y = 5x^2 + 20x + 11$

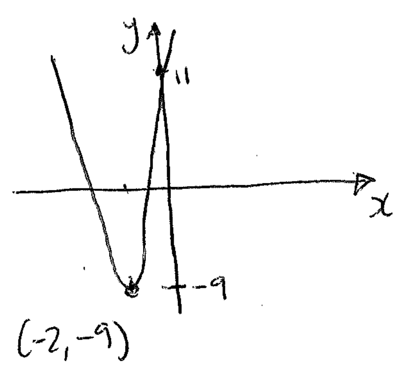
Use formula $x_v = -\frac{b}{2a} = -\frac{20}{2(5)} = \frac{-20}{10} = -2$

$y_v = 5(-2)^2 + 20(-2) + 11$

$= 20 - 40 + 11$

$= -9$ vertex $(-2, -9)$

Range $y \geq -9$



Q6 (d) $y = -x^2 + 10x - 22, \quad a = -1, \quad b = 10, \quad c = -22$

$$x_v = -\frac{b}{2a} = \frac{-10}{2(-1)} = \frac{-10}{-2} = 5$$

$$y_v = -(5^2) + 10(5) - 22$$

$$= -25 + 50 - 22$$

$$= 3$$

Vertex $(5, 3)$ opens downwards (because 'a' is negative)

$y \leq 3$