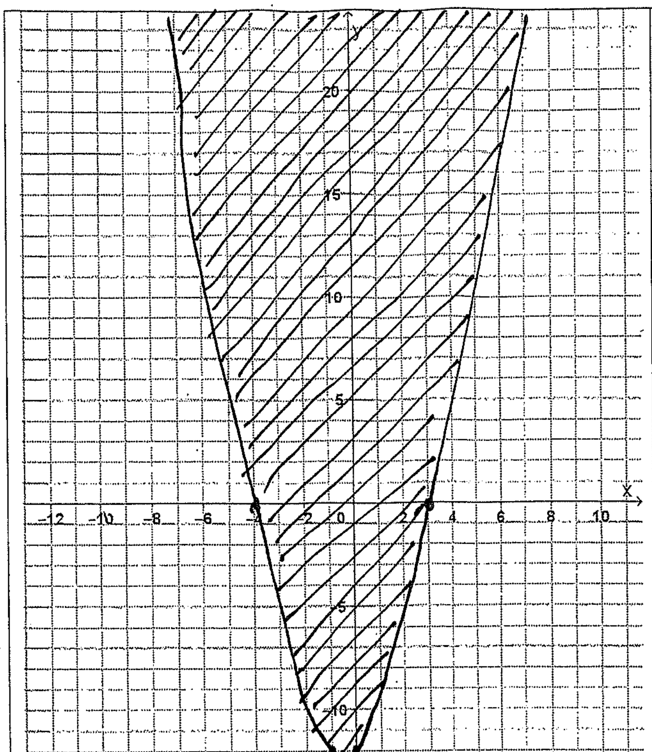
PC Inequalities Assignment 2 version b further practice

This assignment should also serve as a review of unit 4, Quadratics.

- For each straight line, calculate and plot at least three points.
- For each parabola, calculate and plot the vertex, the y-intercept and any x-intercepts.

1,



Shade the region such that:

$$y \geq x^2 + x - 12$$

Find x-intercepts of the parabola by factoring.

parabola $y = x^2 + x - 12$

x int $\Rightarrow y = 0$

$0 = x^2 + x - 12$

$0 = (x+4)(x-3)$

$x = -4$ or $x = 3$

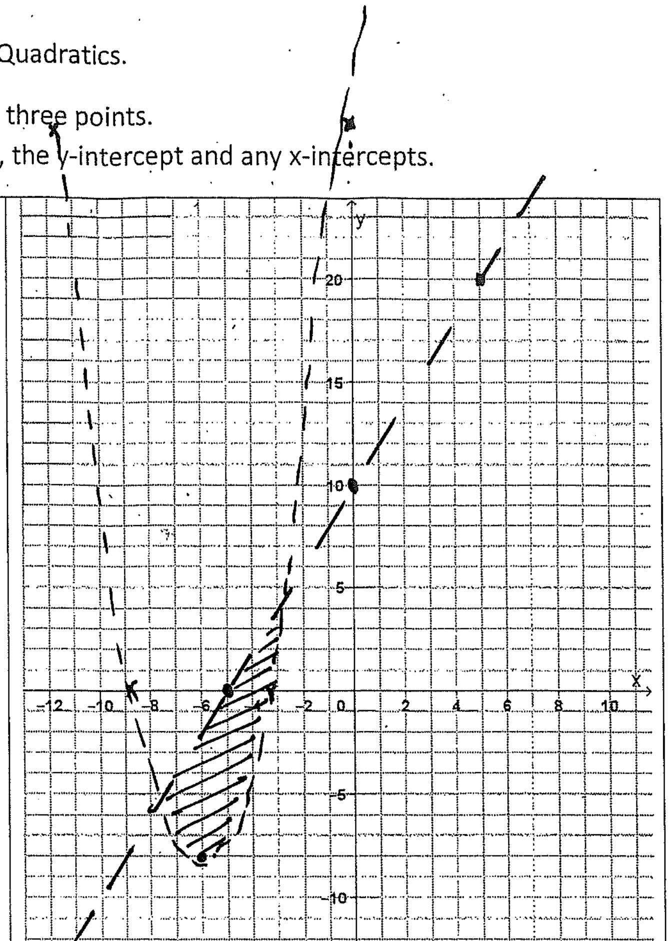
y int $\Rightarrow x = 0$

$y = 0^2 + 0 - 12 = -12$ $(0, -12)$

vertex $x: \frac{-4+3}{2} = \frac{-1}{2}$

$y = (-\frac{1}{2})^2 + (-\frac{1}{2}) - 12 = -12.25$

2,



Shade the region such that:

$$(x+6)^2 - 8 < y < 2x + 10$$

Find x-intercepts of the parabola by rearranging to isolate x.

parabola $y = (x+6)^2 - 8$

vertex $(-6, -8)$

y int $\Rightarrow x = 0$

$y = (0+6)^2 - 8$

$= 36 - 8 = 28$

x int $\Rightarrow y = 0$

$0 = (x+6)^2 - 8$

$8 = (x+6)^2$

$\pm\sqrt{8} = x + 6$

$x_1 = -6 - \sqrt{8} = -8.83$

$x_2 = -6 + \sqrt{8} = -3.17$

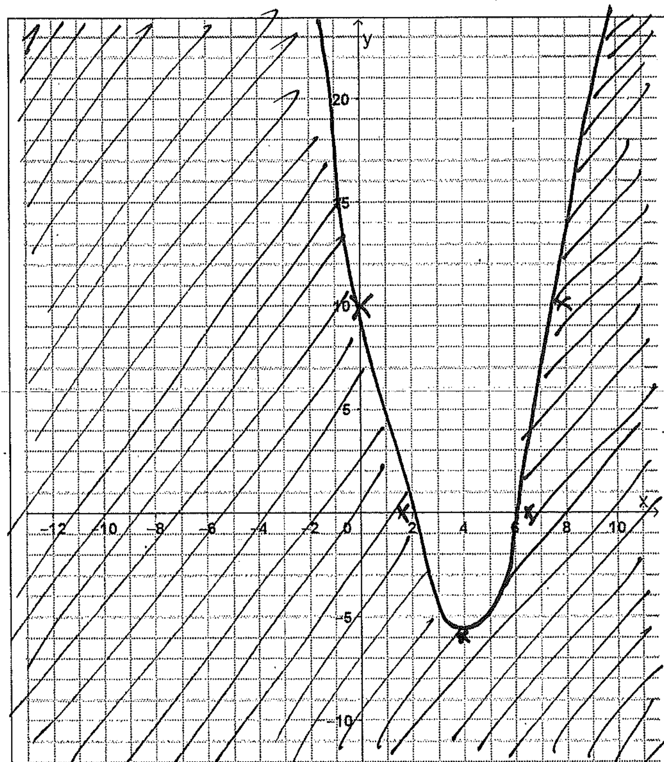
$y = 2x + 10$

$(-5, 0)$

$(0, 10)$

$(5, 20)$

3.



Shade the region such that:

$$y \le x^2 - 8x + 10$$

Find x-intercepts of the parabola by using the quadratic formula.

x int $0 = x^2 - 8x + 10$

$a = 1 \quad b = -8 \quad c = 10$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{8 \pm \sqrt{64 - 4(1)(10)}}{2(1)}$

$= \dfrac{8 \pm \sqrt{24}}{2}$ $\quad x_1 = 1.55$

$x_2 = 6.45$

y int, $x = 0$

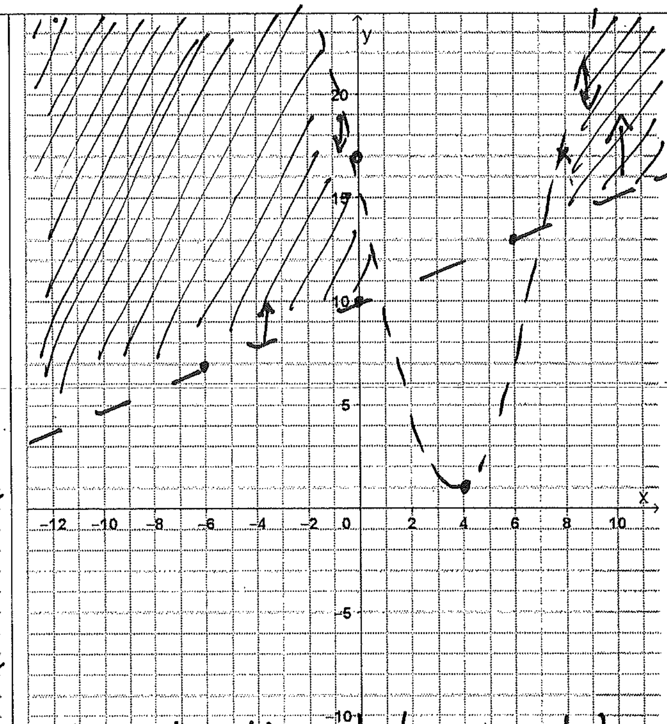
$y = 0^2 - 8(0) + 10 = 10 \quad (0,10)$

vertex, $x = \frac{1.55 + 6.45}{2} = 4$

$y = 4^2 - 8(4) + 10 = -6$

$(4, -6)$

4.



Above the line, below parabola

Shade the region such that:

$$\frac{1}{2}x + 10 < y < (x-4)^2 + 1$$

Find vertex of parabola by completing the square. Then find x-intercepts by rearranging to isolate x.

Already in completed square

Parabola

$y = (x-4)^2 + 1$

vertex $(4, 1)$

x int $y = 0$

$0 = (x-4)^2 + 1$

$-1 = (x-4)^2$

$\sqrt{-1} = x - 4$

no x intercepts, no answer to $\sqrt{-1}$.

y int, $x = 0$

$y = (0-4)^2 + 1$

$= 16 + 1$

$= 17 \quad (0, 17)$

Line

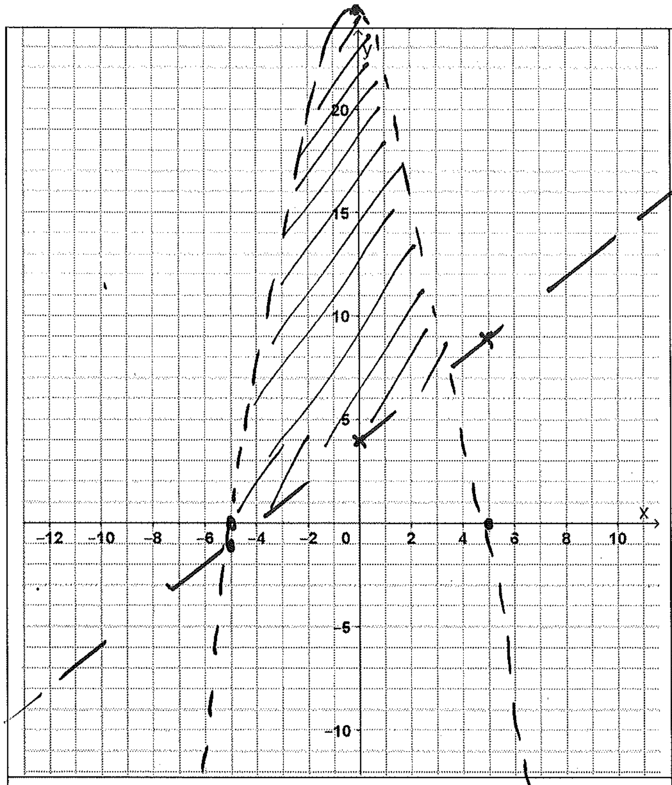
$y = \frac{1}{2}x + 10$

$(-6, 7)$

$(0, 10)$

$(6, 13)$

5.



Shade the region such that:

$$x + 4 < y < 25 - x^2$$

Use any method of calculation (not technology).

$y = x + 4$

$(-5, -1)$

$(0, 4)$

$(5, 9)$

$y = 25 - x^2$

x int, $y = 0$

$0 = 25 - x^2$

$x^2 = 25$

$x = \pm 5$

y int, $x = 0$

$y = 25 - 0^2$

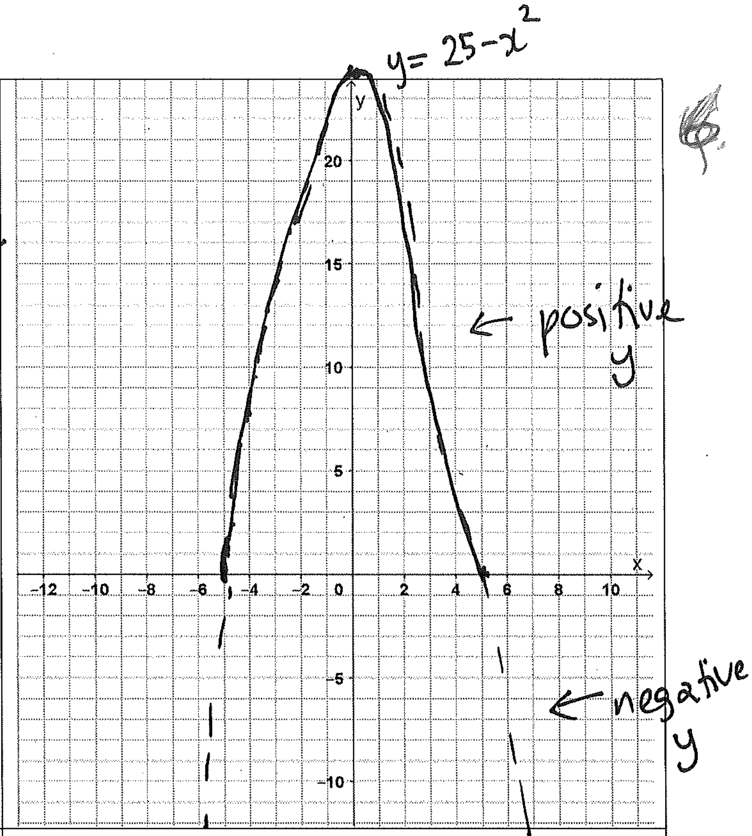
$= 25$

x vertex $= \frac{-5+5}{2} = 0$

y vertex $= 25 - 0^2 = 25$

vertex $(0, 25)$

6.



Calculate the interval of x such that:

$$25 - x^2 > 0$$

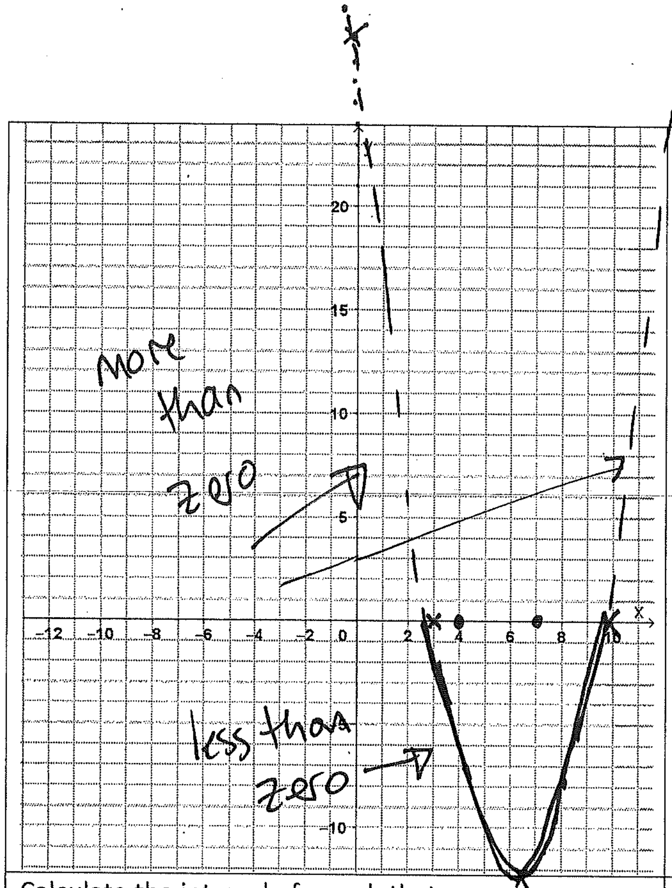
Draw the parabola $y = x^2 - \cancel{9}\,25$. Determine the interval on the x axis for which $x^2 - \cancel{9}\,25$ is negative.

$25 - x^2 > 0$

whenever $\underline{-5 < x < 5}$

(the points on the parabola have a positive y value between $x = -5$ and $x = 5$)

7.



Calculate the interval of x such that:

$$x^2 - 11x + 28 < 2x - 2$$

Method: Rearrange to the form

$$ax^2 + bx + c < 0$$

Sketch the resulting parabola.

~~parabola~~
~~y int (0, 28)~~
~~x int $0 = x^2 - 11x + 28$~~
~~$0 = (x-7)(x-4)$~~
~~$x = 7$ or $x = 4$~~

wrong start

$x^2 - 11x + 28 < 2x - 2$

$\Rightarrow x^2 - 13x + 30 < 0$

$(x-3)(x-10) < 0$

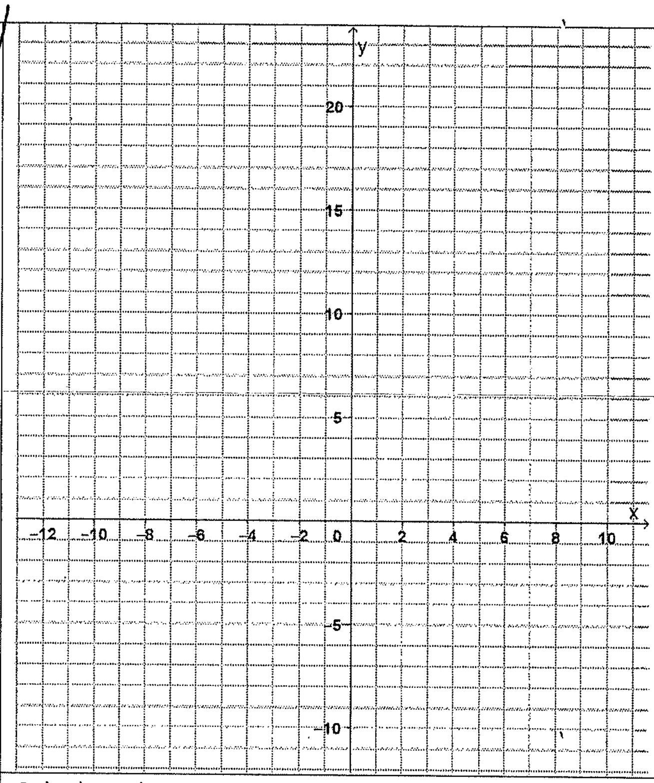
draw $y = x^2 - 13x + 30$

vertex: $x = \frac{10+3}{2} = 6.5$

$y = -12.25$

$3 < x < 10$

8.



Calculate the two intervals of x such that:

$$x^2 - 11x + 28 \geq 2x - 2$$

Method: use your work from the last question.

$x < 3$ or $x > 10$