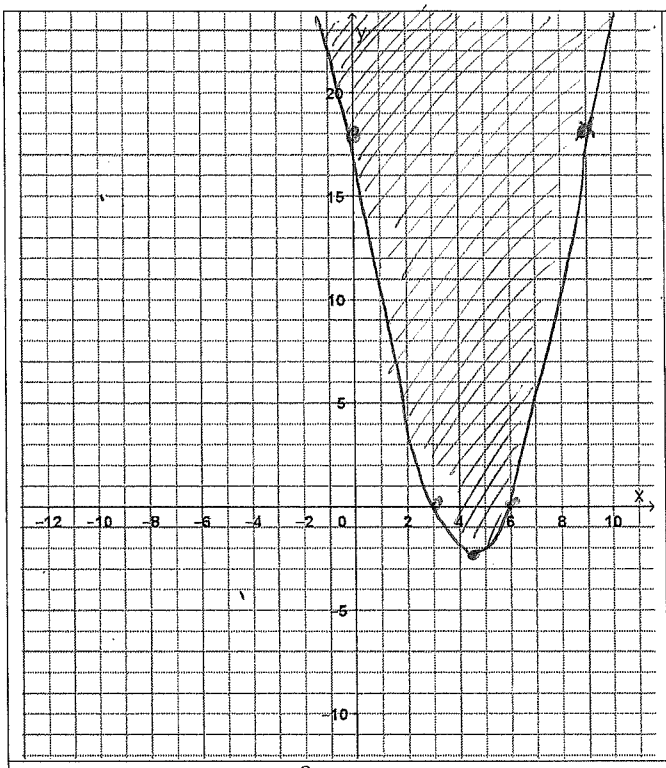
PC Inequalities Assignment 2, 5% Name: Solution

This assignment should also serve as a review of unit 4, Quadratics.

For each straight line, calculate and plot at least three points.

For each parabola, calculate and plot the vertex, the y-intercept and any x-intercepts.

The first five inequalities result in solution regions – shade those. The last three result in solution intervals.



$y \geq x^2 - 9x + 18$

Find x-intercepts by factoring.

$x^2 - 9x + 18$

$= (x-3)(x-6)$

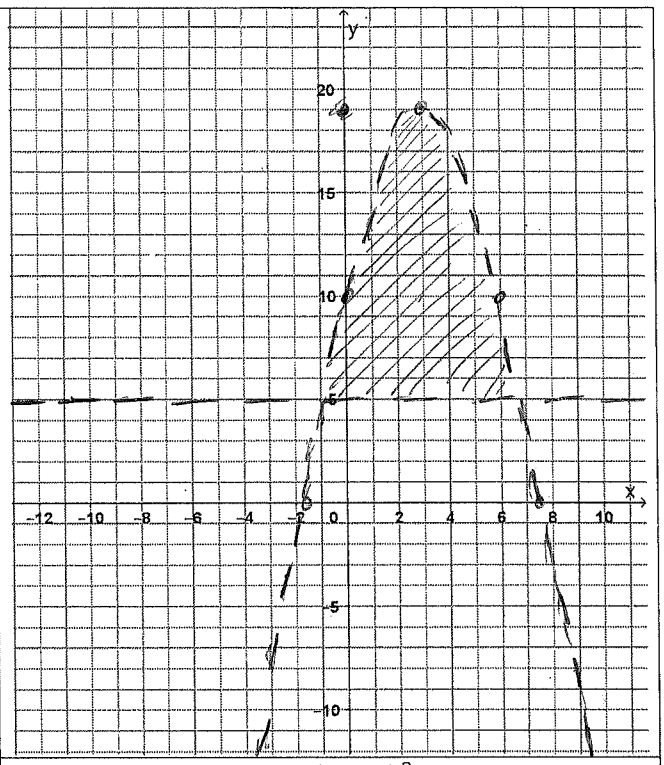
$x-3=0 \qquad x-6=0$

$x=3 \qquad x=6$

$x_v = \frac{3+6}{2} = 4.5$

$y_v = (4.5-3)(4.5-6) = (1.5)(-1.5) = -2.25$

y int $(0, 18)$



$y = 5$

$5 < y < -(x-3)^2 + 19$

Find x-intercepts of the parabola by rearranging to isolate x.

let $y = -(x-3)^2 + 19$

y int, $y = -(0-3)^2 + 19$

$= -9 + 19 = 10$

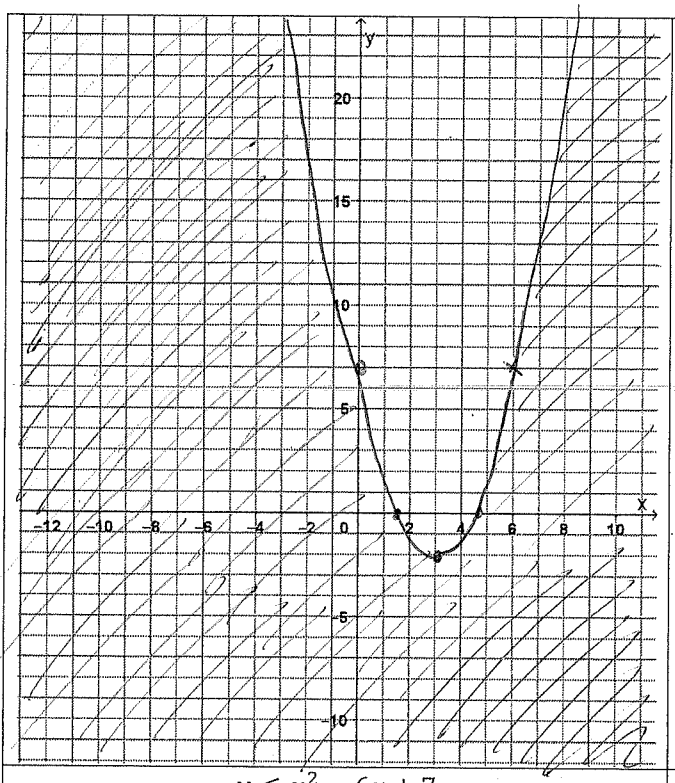
x int, $0 = -(x-3)^2 + 19$

$-19 = -(x-3)^2$

$19 = (x-3)^2$

$\pm\sqrt{19} = x - 3$

$x = 3 \pm \sqrt{19}$, $x = -1.36$, $x = 7.36$



$y \leq x^2 - 6x + 7$

Find x-intercepts using the quadratic formula.

$0 = x^2 - 6x + 7$

$x = \frac{6 \pm \sqrt{36 - 4(1)(7)}}{2}$

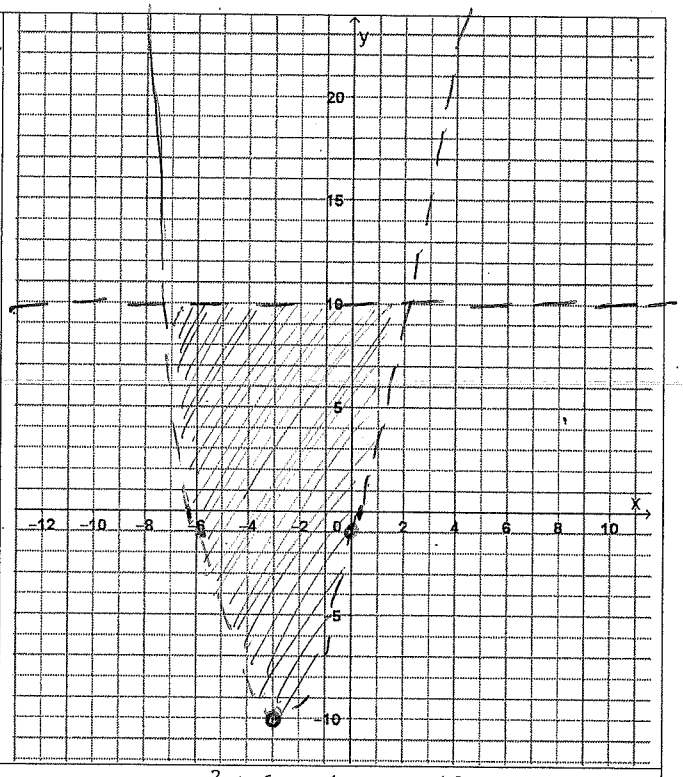
$= \frac{6 \pm \sqrt{8}}{2}$

$x = 1.59, \quad x = 4.41$

$x_v = \frac{1.59 + 4.41}{2} = 3$

$y_v = 3^2 - 6(3) + 7 = -2$

y int $(0, 7)$



$x^2 + 6x - 1 < y < 10$

Find vertex of parabola by completing the square.
Then find x-intercepts by rearranging to isolate x.

$x^2 + 6x - 1$

$= (x+3)^2 - 9 - 1$

$= (x+3)^2 - 10$

Vertex $(-3, -10)$

x int:

$0 = (x+3)^2 - 10$

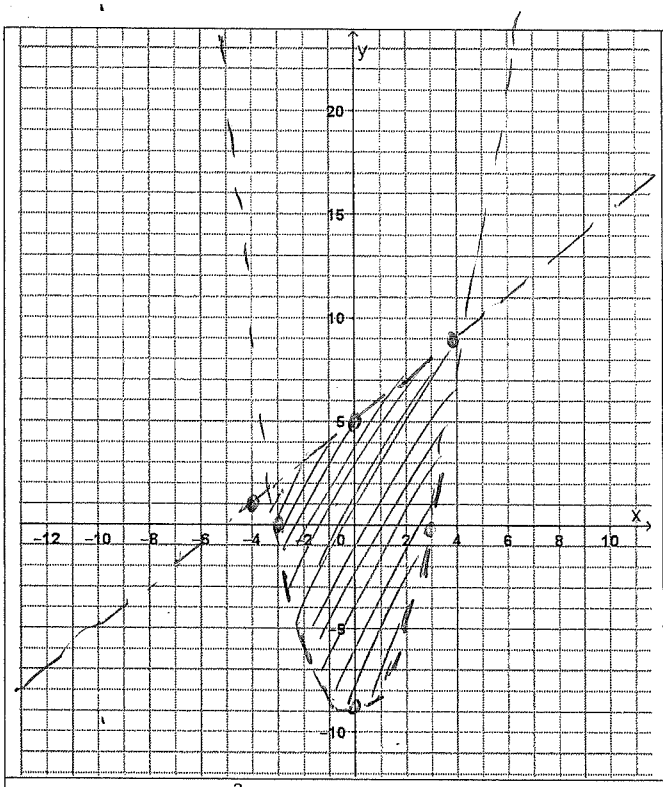
$10 = (x+3)^2$

$\pm\sqrt{10} = x + 3$

$x = -3 \pm \sqrt{10}$

$x = 0.16 \quad x = -6.16$

y int $(0, -1)$



$x^2 - 9 < y < x + 5$

Use any method of calculation (not technology).

Parabola vertex $(0, -9)$

x int: $x^2 - 9 = 0$

$x^2 = 9$

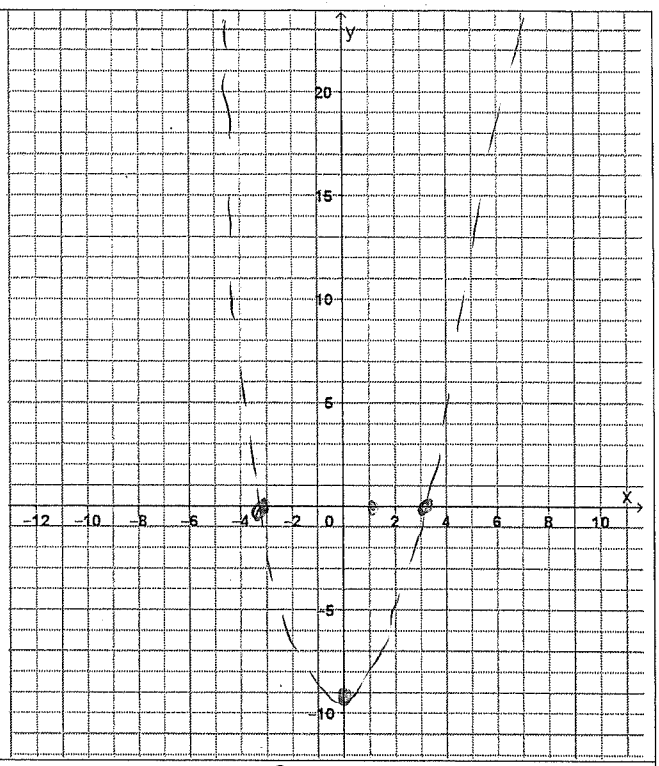
$x = \pm 3$

Line $y = x + 5$

three points $(0, 5)$

$(-4, 1)$

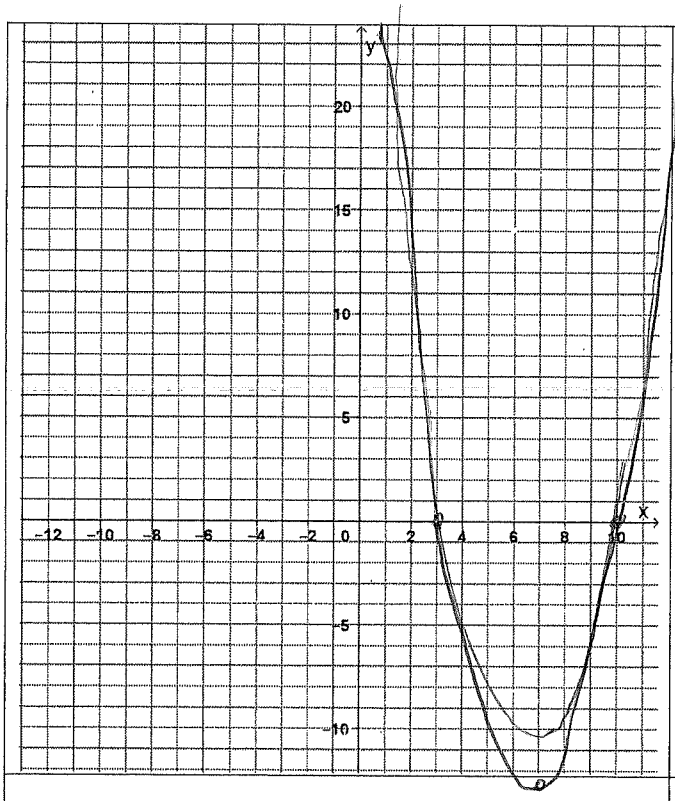
$(4, 9)$

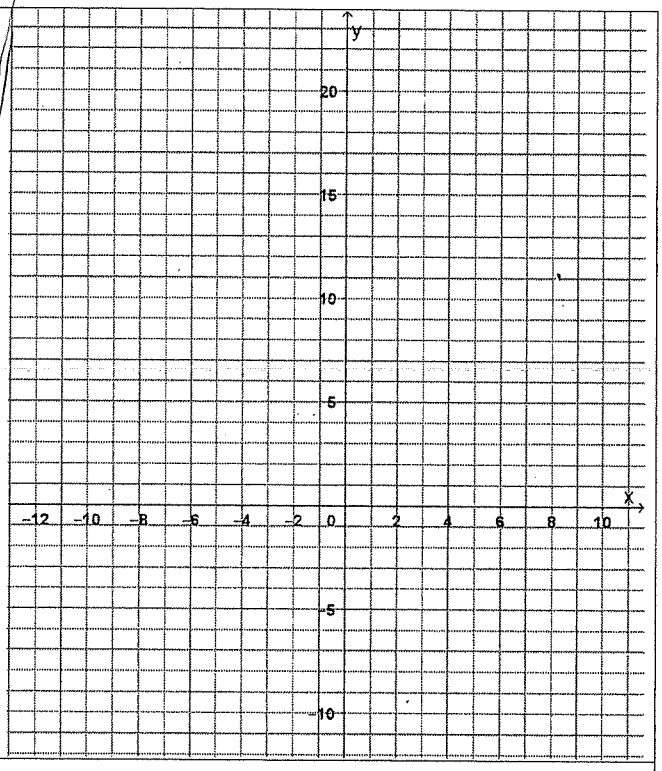


$x^2 - 9 < 0$

Draw the parabola $y = x^2 - 9$. Determine the interval on the x axis for which $x^2 - 9$ is negative.

$-3 < x < 3$





Find the interval of values x such that

$$x^2 - 7x + 12 < 6x - 18$$

Method: Rearrange to the form

$$ax^2 + bx + c \lessgtr 0$$

Sketch the resulting parabola.

$x^2 - 13x + 30 < 0$

Factor

$(x-10)(x-3) < 0$

Critical values $x = 10$, $x = 3$

$3 < x < 10$

Check - let $x = 5$

$5^2 - 7(5) + 12 < ?\ 6(5) - 18$

$25 - 35 + 12 < ?\ 30 - 18$

$2 < 12$ true.

Find the two intervals of values of x such that

$$x^2 - 7x + 12 \geq 6x - 18$$

Method: use your work from the last question.

$x^2 - 7x + 12 \geq 6x - 18$.

$x^2 - 13x + 12 \geq -18$

$x^2 - 13x + 30 \geq 0$

Looking at parabola already drawn

$x^2 - 13x + 30 \geq 0$ when

$x \leq 3$ or $x \geq 10$

Vertex: $x = \frac{10+3}{2} = 6.5$

$y = (6.5-10)(6.5-3) = -3.5^2 = -12.25$