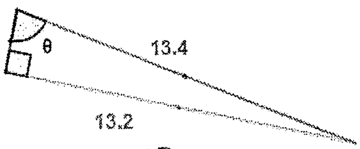
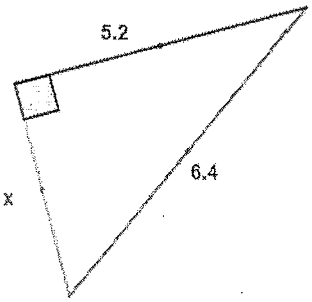
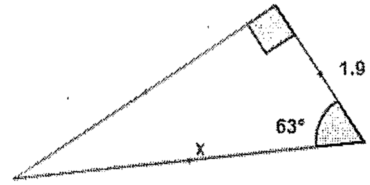
12 Trigonometry Unit Review

1. Right angled triangles. These diagrams are drawn to scale. Make sure your answer makes sense with the diagram.

Calculate the size of the angle marked θ	Calculate the side marked x	Calculate the length of the side marked x
 $\sin\theta = \frac{13.2}{13.4}$ $\theta = \sin^{-1}(13.2 \div 13.4)$ $= 80.1°$	 $x = \sqrt{6.4^2 - 5.2^2}$ $= 3.73$	 $\cos 63 = \frac{1.9}{x}$ $x = \frac{1.9}{\cos 63} = 4.2$

2. Evaluate the exact value of the following trigonometric values. Use trig-diagrams, not a calculator.

$\sin 30°$ $\frac{1}{2}$	$\cos 45°$ $\frac{\sqrt{2}}{2}$	$\tan 60°$ $\sqrt{3}$
$\cos 210°$ $-\frac{\sqrt{3}}{2}$	$\tan 300°$ $-\sqrt{3}$	$\cos(-30)°$ $\frac{\sqrt{3}}{2}$

3 Complete the table of values

Degrees	0	30	90	240	330	45	315	10	40
Radians	0	$\pi/6$	$\frac{\pi}{2}$	$4\pi/3$	$\frac{11\pi}{6}$	$\pi/4$	$7\pi/4$	$\frac{\pi}{18}$	$\frac{4\pi}{18} = \frac{2\pi}{9}$

4. Evaluate the exact value of the following trigonometric values. Use trig-diagrams, not a calculator.

$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$	$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$	$\tan\pi = 0$
$\cos\frac{11\pi}{6} = \frac{\sqrt{3}}{2}$	$\tan\frac{4\pi}{3} = \sqrt{3}$	$\sin(-\frac{\pi}{4}) = -\frac{\sqrt{2}}{2}$

5. Write down at least two trigonometric values that have the following values

1	0	-1	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	undefined
$\sin\pi/2$ $\cos 0$ $\tan\pi/4$	$\sin 0, \sin\pi, \sin 2\pi$ $\cos\frac{\pi}{2}, \cos\frac{3\pi}{2}$ $\tan 0$ $\tan\pi$ $\tan 2\pi$	$\sin\frac{3\pi}{2}$ $\cos\pi$ $\tan\frac{3\pi}{4}$	$\cos\pi/4$ $\sin\pi/4$ $\cos -\pi/4$ $\sin\frac{3\pi}{4}$	$\sin\frac{\pi}{6}$ $\cos\frac{\pi}{3}$	$\tan\frac{\pi}{2}$ $\tan\frac{3\pi}{2}$ $\tan\frac{5\pi}{2}$

$\cos 2\pi$
$\tan\frac{5\pi}{4}$

$\tan\frac{7\pi}{4}$

$\tan\frac{7\pi}{2}$
$\tan -\frac{\pi}{2}$ etc

270

6. Let $-360 \le \theta° \le 720$. Solve:

$\tan\theta = 0.4$	$\cos\theta = -0.3$	$\sin\theta = -1$
-338.2, 21.8, -158 $\theta \in \{-338.2, -158.2, 21.8, 201.8, 381.8, 561.8\}$	107, 253 -253, -107, 107, 253, 467, 613	- 90, 270, 630

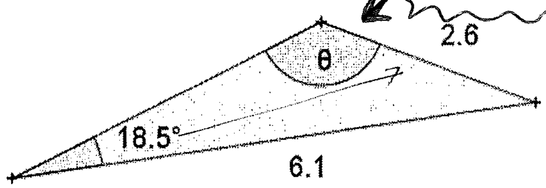
201.8

7. Solve each equation for $\theta \in \mathbb{R}$:

$\tan\theta = 0.1$	$\cos\theta = 0.3$	$\sin\theta = \frac{\sqrt{3}}{2}$
$\theta = 5.7 + 180k$ where $k \in \mathbb{Z}$	~~$\theta_* = \pm 73 + 360k$~~ $\theta = \pm 73 + 360k$, $k \in \mathbb{Z}$	$\theta_1 = 60 + 360k$ $\theta_2 = 120 + 360k$ $k \in \mathbb{Z}$

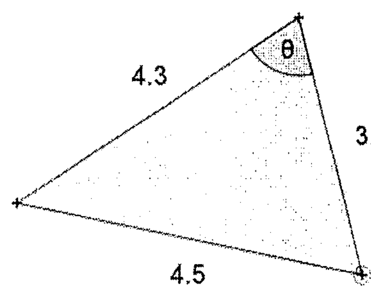
8 Sine Rule, Cosine Rule Calculate the unknown marked with either x or θ in each question. Make sure that your answer makes sense with the diagram.

OBTUSE

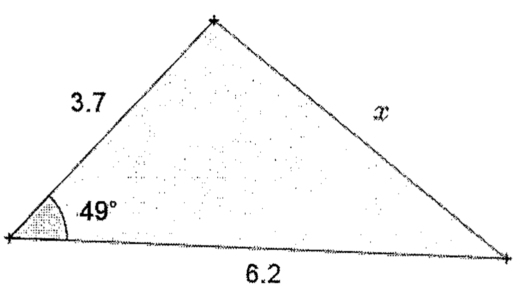


$$\frac{\sin\theta}{6.1} = \frac{\sin 18.5}{2.6}$$

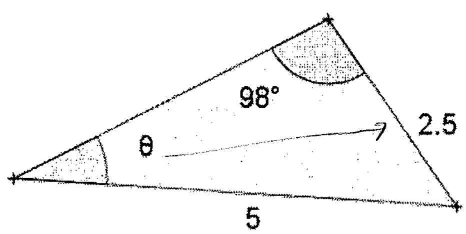
$$\theta = 180 - \sin^{-1}(6.1 \times \sin 18.5 \div 2.6)$$
$$= 132°$$



$$\theta = \cos^{-1}\left(\frac{4.3^2 + 3.3^2 - 4.5^2}{2(4.3)(3.3)}\right)$$
$$= 71°$$



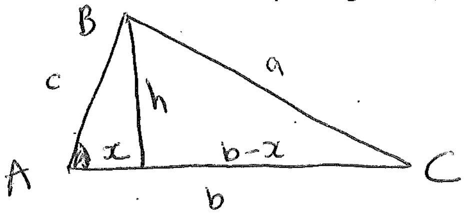
$$x^2 = 3.7^2 + 6.2^2 - 2(3.7)(6.2)\cos 49$$
$$x = \sqrt{ans}$$
$$x = 4.69$$



$$\frac{\sin\theta}{2.5} = \frac{\sin 98}{5}$$

$$\theta = \sin^{-1}(2.5 \times \sin 98 \div 5)$$
$$= \cancel{30°}\ 29.7°$$

9. Show that for any triangle ABC, $a^2 = b^2 + c^2 - 2bc\cos A$



(i) On $\triangle ABC$ introduce height h, so base has two parts x, $b-x$.

(ii) ~~cos~~ $x = c\cos A$ (iii) Two expressions for 'h' using pythagoras.

continued on paper.

10. Show that, if $a^2 = b^2 + c^2 - 2bc\cos A$, then $\cos A = \frac{b^2+c^2-a^2}{2bc}$

$a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 + 2bc\cos A = b^2 + c^2$

$2bc\cos A = b^2 + c^2 - a^2$

finally, divide by $2bc$ to reach

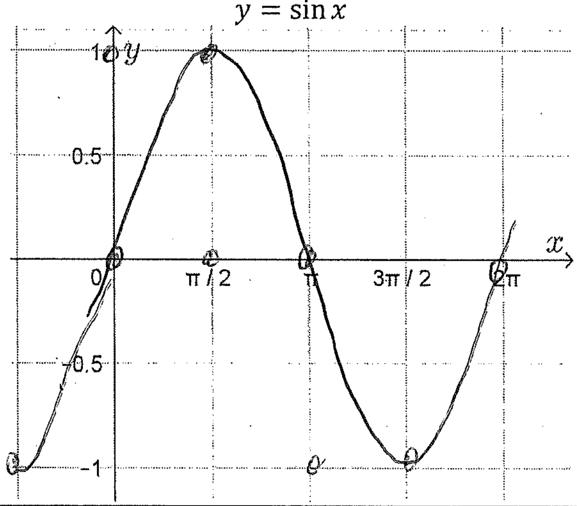
$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$

11. Solve each equation for $0 \le \theta° \le 360$:

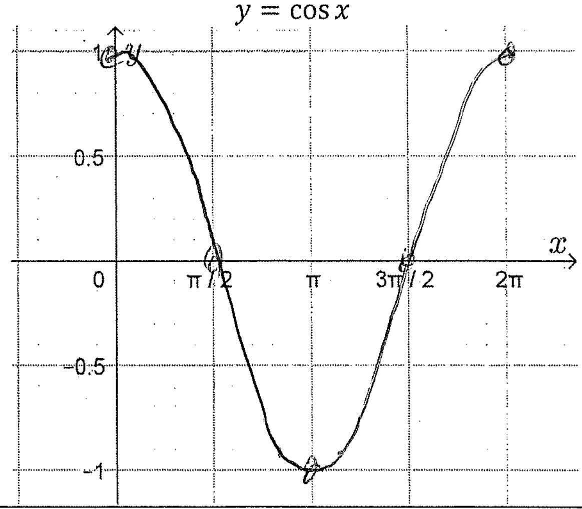
$\sin(2\theta) = 0.5$	$\cos(4\theta) = 0.3$	$2\sin(3\theta) + 6 = 7$
$\sin 30 = 0.5$ $\sin 150 = 0.5$ 360 $\theta_1 = 15$, $\theta_2 = 75$ period=180 $\theta_3 = 195$, $\theta_4 = 255$	8 solutions 360 $\cos 72.5 = 0.3$ $\cos 287.5 = 0.3$ $\theta_1 = 24.2$, $\theta_2 = 95.8$ period $= 90$, so add 90 to each 3 times.	continued on paper.
$\sin(2\theta) = -0.8$	$\cos(3\theta) = -0.7$	$5\sin(2\theta) - 4 = 0$

12. Draw the four basic shapes:

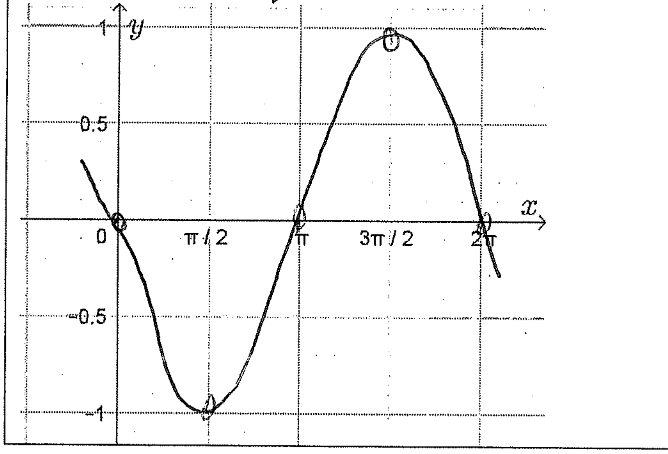
$y = \sin x$



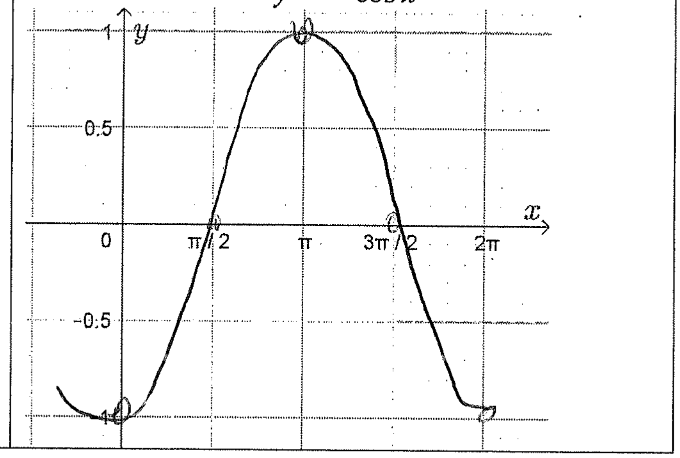
$y = \cos x$



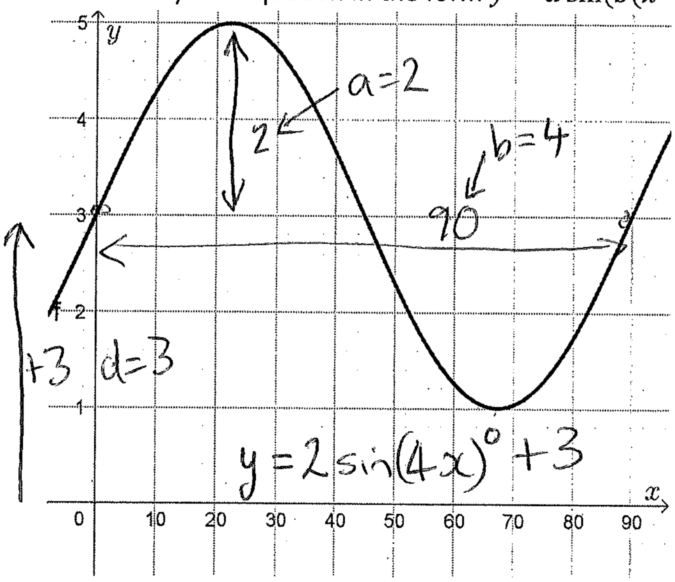
$y = -\sin x$

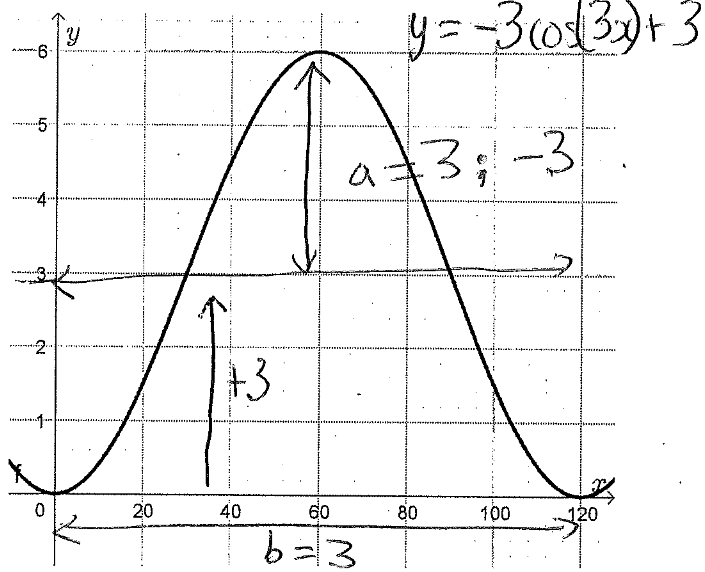


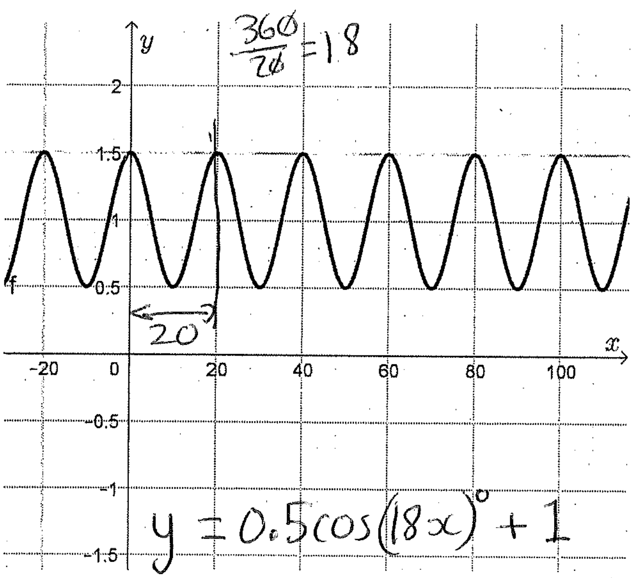
$y = -\cos x$

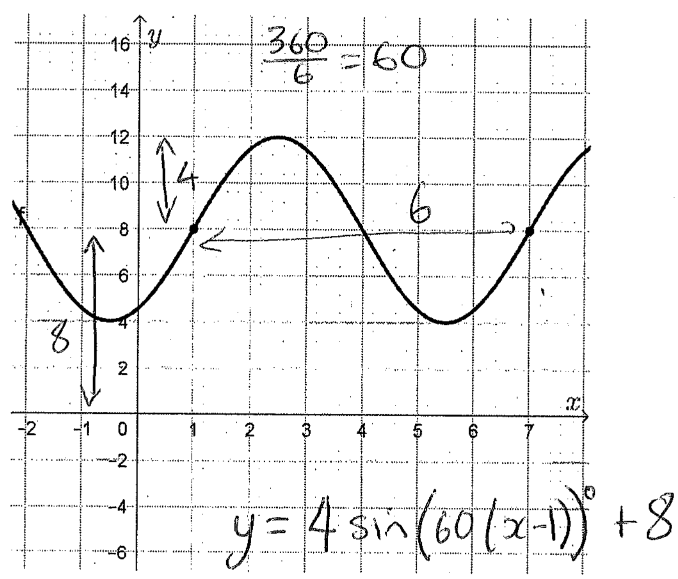


13. Identify the equation in the form $y = a\sin(b(x-c))° + d$, or $y = a\cos(b(x-c))° + d$





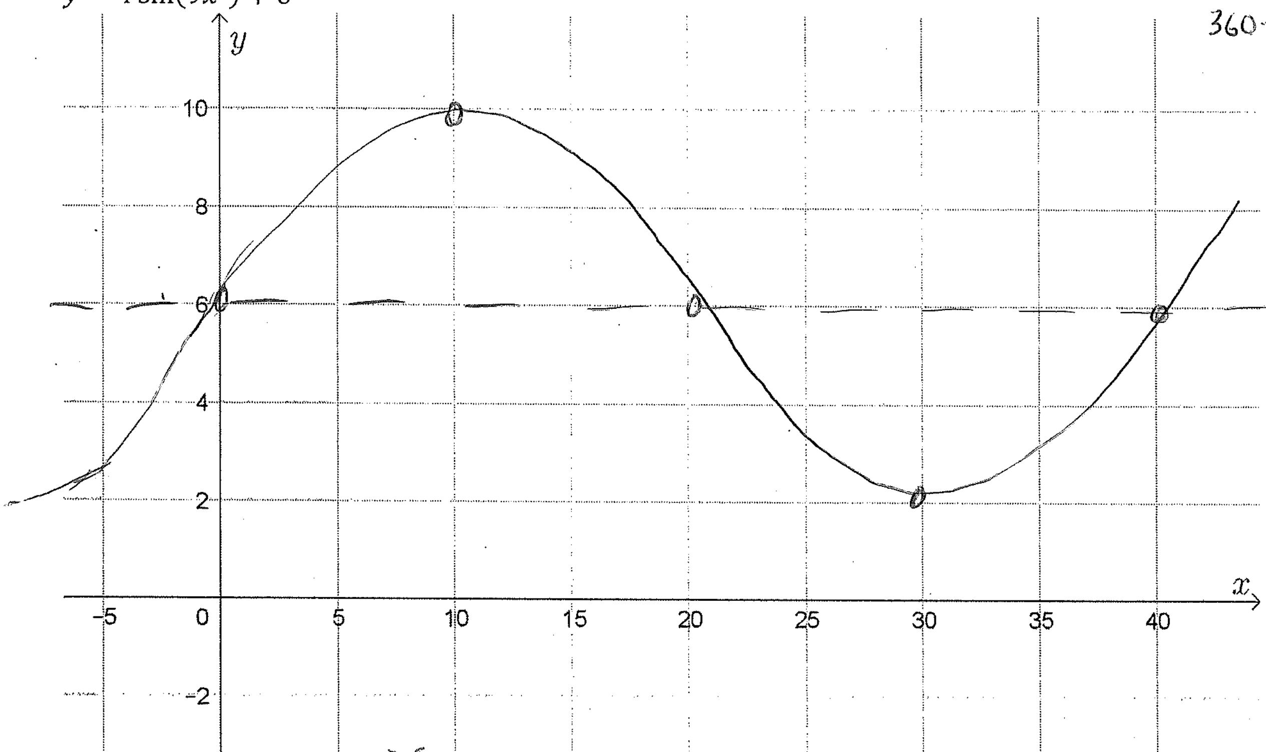




14. Draw the graphs as indicated. Note if the degree symbol is seen, consider the untransformed period to be 360 degrees. Otherwise, by default, the trig calculation is on an angle measured in radians.

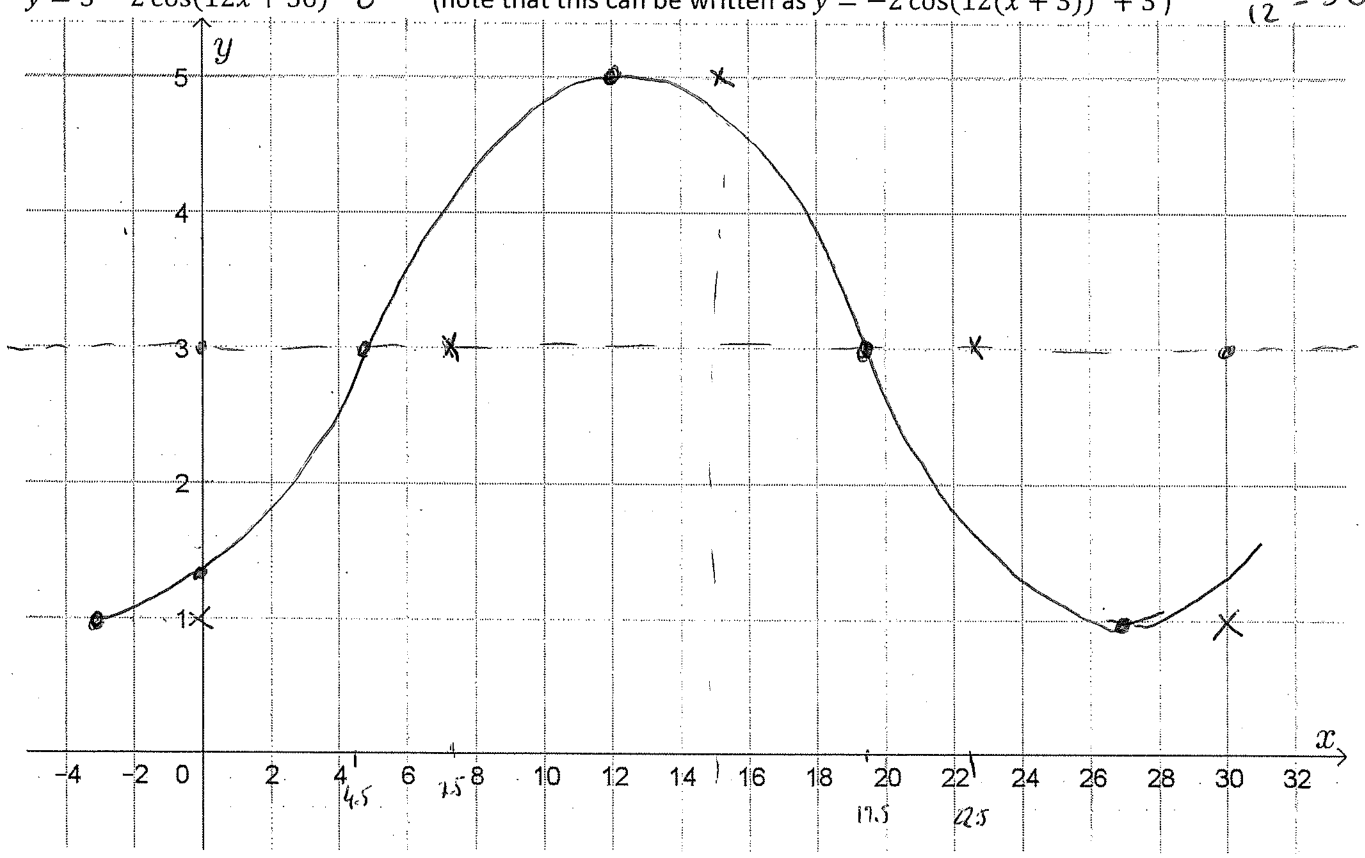
$y = 4\sin(9x^\circ) + 6$

$360 \div 9 = 40$



$y = 3 - 2\cos(12x + 36)^\circ$ (note that this can be written as $y = -2\cos(12(x+3))^\circ + 3$)

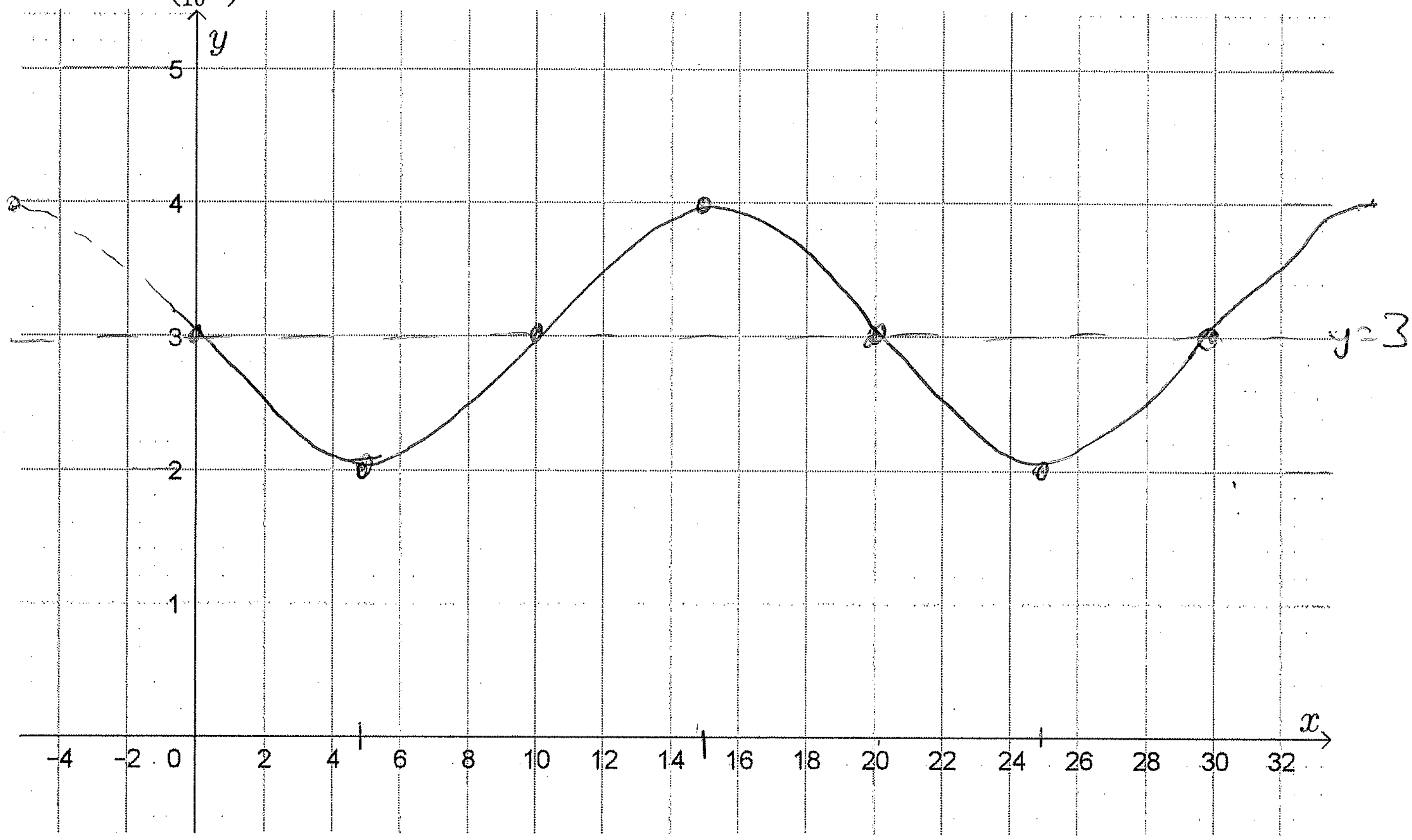
$\frac{360}{12} = 30$



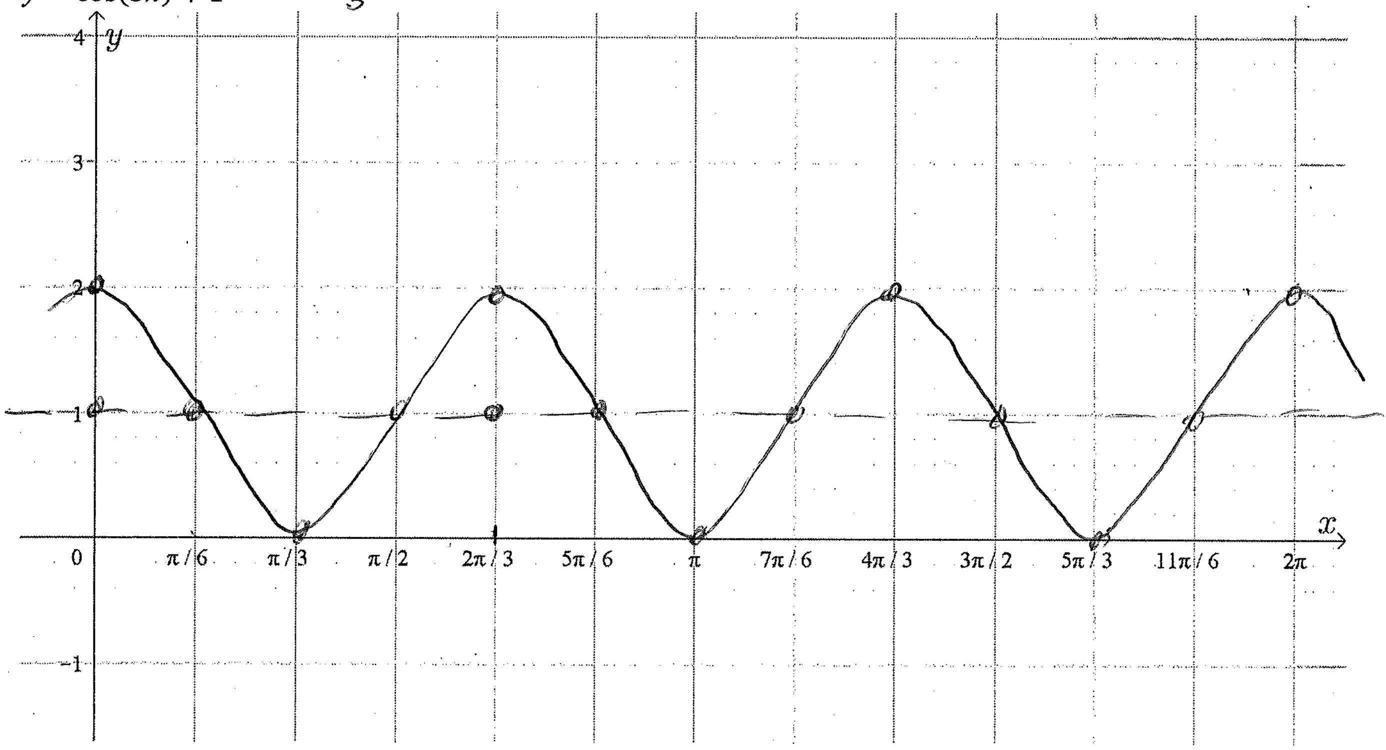
When $x=0$, $-2\cos 36^\circ + 3 = 1.38$

$2\pi \div \frac{\pi}{10} = \frac{2\pi}{1} \times \frac{10}{\pi} = 20$, amp $= 1$, $0, 5, 10, 15, 20$

$y = 3 - \sin\left(\frac{\pi}{10}x\right)$

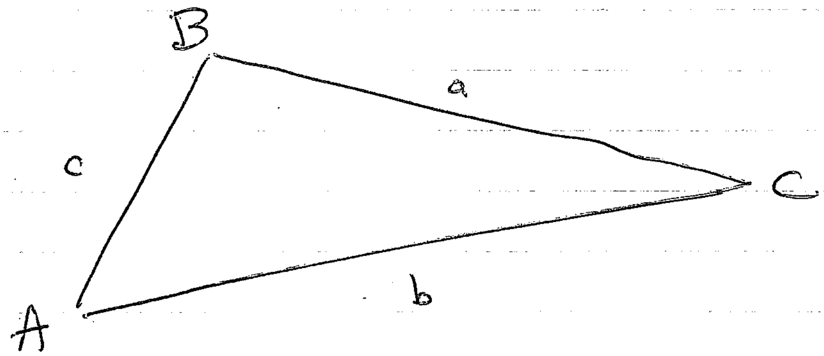


$y = \cos(3x) + 1$ $\frac{2\pi}{3}$



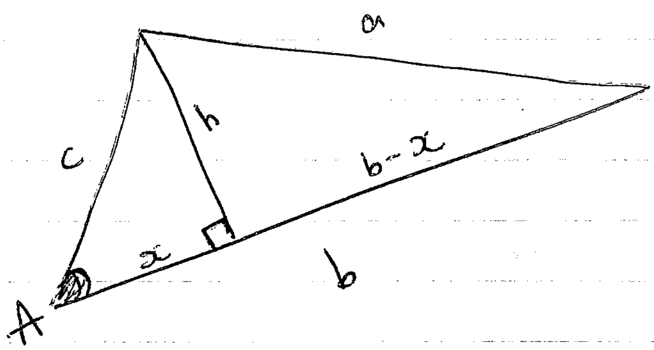
GRADE 12 REVIEW Q 9 + 11

9. Start with triangle ABC (an 'any' triangle)



labelled following convention.

Introduce height 'h' ~~wit~~ which is $\perp$ to base 'b'



Label height 'h'
(or your choice)
label one side of 'b' x,
the other is therefore b-x

(i) $\cos A = \frac{x}{c} \Rightarrow x = c \cdot \cos A$

(ii) $h^2 = \cancel{x^2}\, c^2 - x^2 \qquad h^2 = a^2 - (b-x)^2$

$$\Rightarrow \quad c^2 - x^2 = a^2 - (b-x)^2$$

$$c^2 - x^2 = a^2 - (b^2 - 2bx + x^2)$$

$$c^2 - x^2 = a^2 - b^2 + 2bx - x^2 \quad \text{(add } x^2\text{)}$$

$$c^2 = a^2 - b^2 + 2bx$$

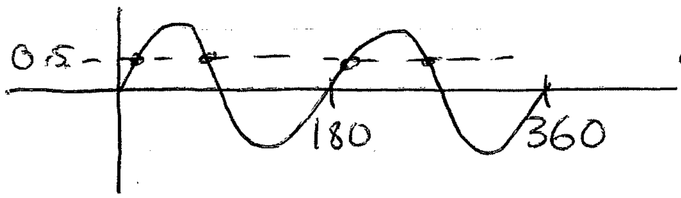
$$b^2 + c^2 - 2bx = a^2$$

replace x with $c \cdot \cos A$

$$b^2 + c^2 - 2bc \cdot \cos A = a^2$$

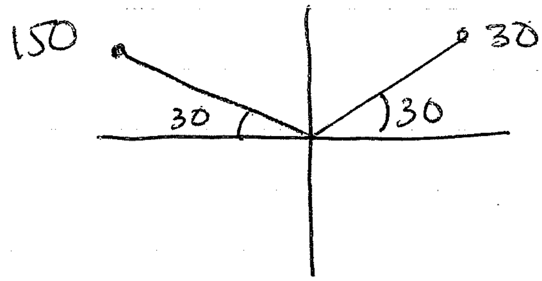
as required.

11.(a) $\sin(2\theta)^\circ = 0.5$ period $= \cancel{90}\ 180$



4 solutions

Now $\sin^{-1}(0.5) = 30^\circ$



we know $\sin 30 = 0.5$

$\sin 150 = 0.5$

but $\sin 2\theta = 0.5$

so $2\theta = 30$, $2\theta = 150$

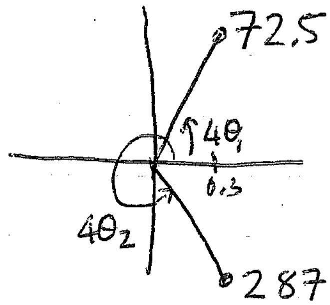
Therefore $\theta_1 = 15$, $\theta_2 = 75$

As the period is 180°, the next solutions are $15 + 180$, $75 + 180$

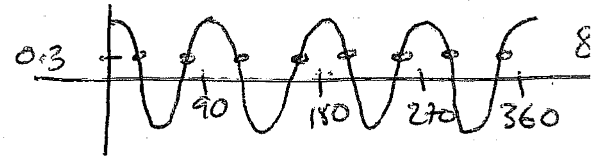
$= 195^\circ$, 255°

Solution set is $\theta \in \{15, 75, 195, 255\}$

(b) $\cos 4\theta = 0.3$ Period $= 360 \div 4 = 90$



$\theta_1 = 18.1$
$\theta_2 = 71.9$



Add period to each of θ_1 and θ_2 until 36

~~$\theta \in \{18.1, \quad , 108.1, \quad , 198.1,$~~

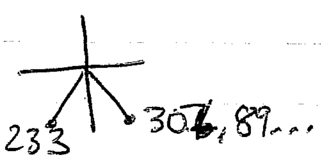
$\theta \in \{18.1, 71.9, 108.1, 161.9, 198.1, 251.9, 288.1, 341.9\}$

(c) $2 \sin 3\theta + 6 = 7$
$2 \sin 3\theta = 1$
$\sin 3\theta = 1/2$ ← nice, $\sin 30 = \frac{1}{2}$, sin 15

$\theta_1 = 10$, $\theta_2 = \cancel{3}50$ period is 120

$\theta \in \{10, 50, 130, 170\ 250, 290\}$ ~~370~~

(d) $\sin 2\theta = -0.8$

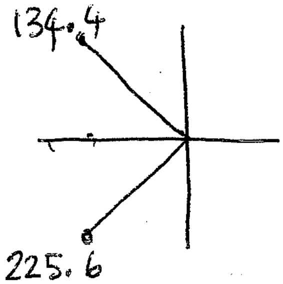


$\theta_1 = 117$, $\theta_2 = 153$

Period $= \cancel{90}\ 180$

$\theta \in \{117, 153, 297, 333\}$

(e) $\cos(3\theta) = -0.7$



$\theta_1 = 44.8$

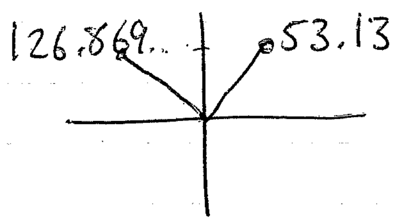
$\theta_2 = 75.2$ Period $= 120$

$\theta \in \{44.8, 75.2, 164.8, 195.2, 284.8, 315.2\}$

(f) $5\sin(2\theta) - 4 = 0$

$5\sin 2\theta = 4$

$\sin 2\theta = 4/5$



$\theta_1 = 26.6$

$\theta_2 = 63.4$ Period $= 180$

$\theta \in \{26.6, 63.4, 206.6, 243.4\}$.