

Solutions

10 FMP Unit 7 Skill 5: Solving Simultaneous Equations with Elimination

Example 1: Same coefficients, same sign

$$\begin{array}{l} \text{(i)} \quad x + 2y = 17 \\ \text{(ii)} \quad x - y = -4 \end{array} \quad \left. \vphantom{\begin{array}{l} \text{(i)} \\ \text{(ii)} \end{array}} \right\}$$

$$+3y = 21$$

$$\boxed{y = 7}$$

$$\begin{array}{l} \text{(ii)} \quad x - y = -4 \\ x - 7 = -4 \\ \boxed{x = 3} \end{array}$$

Check: (i) $x + 2y = 17$
 $3 + 2(7) = ?$
 $3 + 14 = 17 \checkmark$

Example 2: Same coefficients, different sign

$$\begin{array}{l} \text{(i)} \quad 3a + 2b = 13 \\ \text{(ii)} \quad 5a - 2b = -5 \end{array} \quad \left. \vphantom{\begin{array}{l} \text{(i)} \\ \text{(ii)} \end{array}} \right\} \text{ add}$$

$$8a = 8$$

$$\boxed{a = 1}$$

$$\begin{array}{l} \text{(i)} \quad 3a + 2b = 13 \\ 3(1) + 2b = 13 \\ 3 + 2b = 13 \\ 2b = 10 \end{array}$$

$$\boxed{b = 5}$$

Check (ii) $5a - 2b = -5$
 $5(1) - 2(5)$
 $= 5 - 10 = -5 \checkmark$

Example 3: Different coefficients

$$\text{(i)} \quad 2p + q = 1$$

$$\text{(ii)} \quad 3p + 2q = 0$$

$$2 \times \text{(ii)} \quad 6p + 4q = 0 \quad \left. \vphantom{\begin{array}{l} \text{(ii)} \\ 2 \times \text{(ii)} \end{array}} \right\} \text{ subtract}$$

$$\boxed{p = 2}$$

from (i) $2p + q = 1$

$$2(2) + q = 1$$

$$4 + q = 1$$

$$\boxed{q = -3}$$

Check: (ii) $3p + 2q = 0$
 $3(2) + 2(-3)$
 $= 6 - 6 = 0 \checkmark$

Example 4: Different coefficients

$$\text{(i)} \quad 5m + 2n = 6 \quad \times 4$$

$$\text{(ii)} \quad 3n + 4m = -4$$

$$\text{reorder (ii)} \quad 4m + 3n = -4 \quad \times 5$$

$$4 \times \text{(i)} \quad 20m + 8n = 24$$

$$5 \times \text{(ii)} \quad 20m + 15n = -20$$

} subtract

$$7n = -44$$

$$\boxed{n = -44/7}$$

$$\text{(i)} \quad 5m + 2\left(-\frac{44}{7}\right) = 6$$

$$35m - 88 = 42$$

$$35m = 130$$

$$\boxed{m = \frac{130}{35} = \frac{26}{7}}$$

Check: (ii) $3\left(-\frac{44}{7}\right) + 4\left(\frac{26}{7}\right)$

$$= -132/7 + 104/7 = -28/7 = -4 \checkmark$$

Practice:

1, (i) $a+b=12$
 (ii) $2a-b=3$ } add

$$3a = 15$$

$$\boxed{a = 5}$$

(i) $a+b=12$
 $5+b=12$
 $\boxed{b=7}$

check: (ii) $2(5)-7=3 \checkmark$

2, (i) $3m-2n=5$
 (ii) $3m+n=8$ } subtract

$$-3n = -3$$

$$\boxed{n=1}$$

(ii) $3m+n=8$
 $3m+1=8$
 $3m=7$
 $\boxed{m=7/3}$

Check (i) $3 \cdot \frac{7}{3} - 2(1) = 7 - 2 = 5 \checkmark$

3, (i) $5p-2q=2$
 (ii) $5p+4q=56$ } subtract

$$-6q = -54$$

$$\boxed{q=9}$$

(i) $5p-2q=2$
 $5p-18=2$
 $5p=20$
 $\boxed{p=4}$

check (ii) $5(4)+4(9)=20+36=56 \checkmark$

4, $x-y=1$
 $x+y=1$ } add

$$2x = 2$$

$$\boxed{x=1}$$

$$y=0$$

easy enough to check in both!

5, $a+2b=1$
 $a-2b=-7$ } subtract
 add

$$2a = -6$$

$$\boxed{a = -3}$$

(i) $a+2b=1$
 $-3+2b=1$
 $2b=4$
 $\boxed{b=2}$

(ii) $(-3)-2(2)=-3-4=-7 \checkmark$

6, $3m+4n=8$ $\times 2$
 $2m+3n=7$ $\times 3$

$$6m+8n=16$$

$$6m+9n=21$$
 } subtract

$$\boxed{n=5}$$

(i) $3m+4n=8$
 $3m+20=8$
 $3m=-12$
 $\boxed{m=-4}$

(ii) $2(-4)+3(5)=-8+15=7 \checkmark$

7.

$$\begin{aligned} p - 2q &= -14 \\ q + 2p &= 22 \end{aligned}$$

reorder

$$\begin{aligned} p - 2q &= -14 && \times 2 \\ 2p + q &= 22 && \} \text{ subtract} \\ 2p - 4q &= -28 && \} \end{aligned}$$

$$5q = 50$$

$$\boxed{q = 10}$$

$$(i) p - 2q = -14$$

$$p - 20 = -14$$

$$\boxed{p = 6}$$

$$(ii) q + 2p = 10 + 12 = 22 \checkmark$$

8.

$$\begin{aligned} x - y &= 16 \\ x + y &= 224 \end{aligned}$$

} add

$$2x = 240$$

$$\boxed{x = 120}$$

$$(ii) x + y = 224$$

$$120 + y = 224$$

$$\boxed{y = 104}$$

$$(i) 120 - 104 = 16 \checkmark$$

9.

(reorder)

$$\begin{aligned} 3a - 4b &= 45 && \times 7 \quad 5 \\ 5b - 7a &= 25 && \times 4 \end{aligned}$$

$$\begin{aligned} 21a - 28b &= 315 \\ 20 \quad 20b &= 10 \text{ acht...} \end{aligned}$$

$$\begin{aligned} 15a - 20b &= 225 && \} \text{ add} \\ -28a + 20b &= 100 && \} \end{aligned}$$

$$-13a = 325$$

$$\boxed{a = -25} \quad \boxed{b = -30}$$

$$(ii) 5(-30) - 7(-25) = -150 + 175 = 25 \checkmark$$

10.

$$\begin{aligned} 2m + 7n &= 55 && \times 3 \\ 3m + 2n &= 6 && \times 2 \end{aligned}$$

$$\begin{aligned} 6m + 21n &= 165 \\ 6m + 4n &= 12 && \} \text{ subtract} \end{aligned}$$

$$17n = 153$$

$$\boxed{n = 9}$$

$$(i) 2m + 5n = 55$$

$$\boxed{m = -4}$$

$$(ii) 3(-4) + 2(9) = -12 + 18 = 6 \checkmark$$

11.

$$\begin{aligned} 12p - q &= 38 && \times 3 \\ 2p + 3q &= 19 && \} \text{ add} \end{aligned}$$

$$36p - 3q = 114$$

$$38p = 133$$

$$p = 3.5$$

$$(i) 12(3.5) - q = 38$$

$$q = 4$$

$$(ii) 2(3.5) + 3(4) = 19 \checkmark$$

12.

$$\begin{aligned} 11x + 11y &= 12 && \times 7 \\ 7x + 6y &= 7 && \times 11 \end{aligned}$$

$$\begin{aligned} 77x + 77y &= 84 \\ 77x + 66y &= 77 && \} \text{ subtract} \end{aligned}$$

$$11y = 7$$

$$y = \frac{7}{11}$$

$$(i) 11x + 11 \cdot \frac{7}{11} = 12$$

$$11x + 7 = 12$$

$$11x = 5$$

$$x = \frac{5}{11}$$

$$(ii) 7\left(\frac{5}{11}\right) + 6\left(\frac{7}{11}\right) = \frac{35 + 42}{11} = \frac{77}{11} = 7 \checkmark$$

Problem Solving

Choose two letters to represent the two unknowns in each problem. Set up two equations about the two unknowns using the information given and solve. Use any method to solve.

Two consecutive integers add up to 37. What are they?

$$\begin{aligned} a + b &= 37 \\ a - b &= 1 \end{aligned} \left. \vphantom{\begin{aligned} a + b &= 37 \\ a - b &= 1 \end{aligned}} \right\} \text{add}$$

$$2a = 38 \quad a = 19, \quad b = 18$$

The sum of two integers is 150. Their difference is 28. What are they?

$$\begin{aligned} a + b &= 150 \\ a - b &= 28 \end{aligned} \left. \vphantom{\begin{aligned} a + b &= 150 \\ a - b &= 28 \end{aligned}} \right\} \text{add}$$

$$2a = 178 \quad b = 61$$

$$a = 89$$

There are 105 sheep (ewes) and lambs in a field. Every ewe has two lambs. How many ewes and how many lambs are there?

(i) $2e = l \Rightarrow \cancel{2e = l = 0}$

substitute, (ii) $e + l = 105$

$$e + 2e = 105$$

$$3e = 105$$

$$e = 35, \quad \underline{l = 70}$$

The teacher goes to Red Barn to buy 30 bottled drinks, water and fruit juice, for the class as its very hot in the classroom during the summer. The total cost is \$44.10. The water costs \$0.75 each; the juice is \$1.95 each. How many water and how many juice did they buy?

$$\begin{aligned} w + j &= 30 \\ 0.75w + 1.95j &= 44.10 \end{aligned} \left. \vphantom{\begin{aligned} w + j &= 30 \\ 0.75w + 1.95j &= 44.10 \end{aligned}} \right\} \text{subtract}$$

$$0.75w + 0.75j = 22.5$$

$$1.2j = 21.6$$

$$j = \frac{21.6}{1.2} = 18, \quad 12 \text{ water}$$

A line has general equation $y = mx + b$.

A line goes through points (3, 8) and (10, 43). This yields the following two equations:

$$\begin{aligned} 8 &= 3m + b \\ 43 &= 10m + b \end{aligned} \left. \vphantom{\begin{aligned} 8 &= 3m + b \\ 43 &= 10m + b \end{aligned}} \right\} \text{subtract}$$

Solve the equations to find the slope m and y intercept b , and therefore the equation of the line:

$$35 = 7m$$

$$\boxed{m = 5} \quad 8 = 15 + b \quad \boxed{b = -7} \quad \begin{matrix} y = 5x \\ y = 5x - 7 \end{matrix}$$

A line goes through points (2, 25) and (7, 60). Use simultaneous equations to find the equation of the line.

$$\begin{aligned} 25 &= 2m + b \\ 60 &= 7m + b \end{aligned} \left. \vphantom{\begin{aligned} 25 &= 2m + b \\ 60 &= 7m + b \end{aligned}} \right\} \text{subtract}$$

$$35 = 5m \quad \boxed{y = 7x + 11}$$

$$m = 7$$

A parabola has general equation $y = ax^2 + bx + c$.

A parabola goes through points (3, 7); (4, 16) and (5, 27). Set up three equations, using the x and y coordinates given. Figure out the values a , b and c using the process of elimination a few times over. Try out your solution on geogebra to see if your solution goes through the three points given.

$$\begin{aligned} 7 &= 9a + 3b + c \\ 16 &= 16a + 4b + c \\ 27 &= 25a + 5b + c \end{aligned} \left. \vphantom{\begin{aligned} 7 &= 9a + 3b + c \\ 16 &= 16a + 4b + c \\ 27 &= 25a + 5b + c \end{aligned}} \right\} \text{eliminate } c$$

$$\begin{aligned} 9 &= 7a + b \\ 11 &= 9a + 2b \end{aligned} \left. \vphantom{\begin{aligned} 9 &= 7a + b \\ 11 &= 9a + 2b \end{aligned}} \right\} 2 \otimes = 2a$$

$$a = 1 \quad b = 2$$

$$7 = 9(1) + 3(2) + c$$

$$7 = 9 + 6 + c$$

$$7 = 15 + c$$

$$c = -8$$

$$y = x^2 + 2x - 8$$

check,

$$\begin{aligned} x=4: & 16 + 8 - 8 = 16 \checkmark \\ x=5: & 25 + 10 - 8 = 27 \checkmark \\ x=3: & 9 + 6 - 8 = 7 \checkmark \end{aligned}$$